'Shine like stars in the world.'
Philippians 2:15

St Paul's CE Primary School

Maths Written-Calculation Policy

Vision

Our school is one family, united in love and deeply rooted in our Christian values, where together on life's journey we flourish, striving for excellence in all that we do. Inspired by the transformation of St Paul, and enlightened by the glory of God, we will shine like stars to make the world a better place.

Values

Love, Forgiveness, Faith, Friendship, Hope and Peace.

Rationale

This policy has been designed to show progression in written mathematical methods throughout the school. Our written calculation policy is set out to show:

- The objectives stipulated for all four operations by the National Curriculum
- The calculation methods for each year group
- Relevant vocabulary needed at each stage
- Examples of reasoning activities

Concrete, Pictorial and Abstract

Each method has examples as to what it looks like in the concrete, pictorial and abstract forms. All learners from EYFS to Year 4 are introduced to a calculation method for the first time using concrete manipulatives. Concrete resources from EYFS to Year 6 include: bead strings, Dienes, Cuisenaire Rods, Place Value counters, Numicon and multilink cubes. Children will then progress through to a pictorial stage before moving to the abstract. During the pictorial stage, children will be taught to use the bar model (see separate guidance booklet). The amount of time needed to progress through each stage is unique to each learner.

Mastering Calculation

The new curriculum has a strong focus on mastery and therefore, if a child is fluent in a method for their year group, they should not (normally) be moved onto a different method of calculation. Instead, children will be encouraged by their teacher to 'go deeper' within this method. This may involve: using it in different contexts; using and applying it to other learning; using it with missing digits or values; explaining or experimenting with different aspects of it; proving answers with pictures or manipulatives; or explaining what has gone wrong in a calculation. Children must also check their calculations through the use of estimation and inverse operations.

Mathematical Vocabulary

The National Curriculum places great emphasis upon the use of correct mathematical vocabulary and children developing this. Throughout school, children are strongly recommended to use and apply mathematical vocabulary when learning a new method or concept. They will be constantly exposed to this, have it expertly modelled by their teacher and be expected to use it themselves when justifying methods.

Mental Methods

Children should always be encouraged to see if they can work out a calculation mentally before trying a written method.
Children will be shown number patterns and relationships between numbers throughout the school. Times tables are introduced and taught in specific year groups:
Reception: x2;
Year 1: x5, x10
Year 2: x4, x3
Year 3: x6, x8, x7, x9, x11, x12.
Year 4: consolidation

Mathematics Educational Programme

Developing a strong grounding in number is essential for providing children with the platform to excel mathematically. Children should develop a deep conceptual understanding of the numbers to 10, the relationships between them and the patterns therein. By providing frequent and varied opportunities to build and apply this understanding, children will develop a secure base of knowledge from which mathematical mastery is built.

EYFS ELG Number:	Children at the expected level of development will: - Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order; - Subitise (recognise quantities without counting) up to 5; - Automatically recall number bonds for numbers 0-5 and for 10, including corresponding partitioning facts.
EYFS ELG Numerical Patterns:	Children at the expected level of development will: - Automatically recall double facts up to 5+5; - Compare sets of objects up to 10 in different contexts, considering size and difference; - Explore patterns of numbers within numbers up to 10, including evens and odds.

Key Stage 1 - Year 1

	+	-	x	÷
National Curriculum Objective	• Read, write and interpret statements involving addition, subtraction and equals sign. • Add and subtract 1 and 2 digit numbers to 20, including zero. • Represent and make number bonds and related subtraction facts within 20. • Regroup to 10 to make 10.		• Double and halve numbers to 10 through grouping and sharing. • Make links to counting in multiples of 2, 5 and 10 – drawing arrays. • Reason about odd and even numbers and relate to doubling and halving. • Solve one-step problems involving multiplication and division. • Share objects into equal sized groups.	
Suggested calculation	• Counting on using number lines and number tracks • Informal partitioning	Counting back	Repeated addition and arrays	Sharing and grouping
Mathematical vocabulary	count on, count back, number bonds, number facts, subtraction facts, fact family, add, subtract, more, less, plus, minus, total, sum, difference between, equal		grouping, sharing, multiply, divide, double, half, array, lots of	

Year 2

	+	-	x	÷
National Curriculum Objective	• Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 • Understand the = sign and how 20 + 2 and 24 – 2 both have the same value of 22. • Add and subtract numbers: - a 2-digit number and ones (no regrouping & regrouping in the ones) - a 2-digit number and tens (no regrouping & regrouping in the ones)		• Introduction of arrays in a grid method. • Write mathematical statements using the multiplication (×), division (÷) and equals (=) signs. • Link multiplication and division through missing number questions. • Make links to counting in multiples of 4 and 8. • Share and group objects. • Group using repeated subtraction.	
Suggested calculation	• Informal partitioning • Partitioning column • Column method		• Arrays in a grid	• Sharing and grouping in arrays
Mathematical vocabulary	Add, subtract, count on, count back, more, less, plus, minus, total, sum, difference, partition, bridge, round, inverse, number line, number facts, multiple of 10, regroup		Inverse, operation, multiplication table, times table, multiply, multiplication, times, product, repeated addition, lots of, array, divide, division, shared by, halve, double	

Lower Key Stage 2 - Year 3

	+	-	x	÷
National Curriculum Objective	• Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100. • Add and subtract numbers: - up to three digits - two 2 digit numbers - adding 3 one digit numbers		• Multiply 2 digit numbers by a 1 digit. • Introduction of the grid method. • Make links to counting in multiples of 3,6,9. • Share using place value headings TU ÷ U. • Introduce remainders.	
Suggested calculation	column method		• Short multiplication	• Sharing and grouping within place value columns •
Mathematical vocabulary	Add, subtract, count on, count back, more, less, plus, minus, total, sum, difference, partition, bridge, round, inverse, number facts, multiple of 10, regroup		Inverse, operation, multiplication table, times table, multiply, multiplication, times, product, repeated addition, lots of, array, divide, division, shared by, halve, double	

Year 4

	+	-	x	÷
National Curriculum Objective	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction.		• Multiply 2/3 digits by a 1-digit number. • Recall multiplication & division facts up to 12 x 12.	Divide numbers up to 3 digits by a 1 digit number using the formal written method of short division and interpret remainders appropriately in context
Suggested calculation	Compact column		• short multiplication	• short division
Mathematical vocabulary	addition, subtraction, sum, total, difference, minus, less, plus, altogether, column addition, column subtraction, regroup, operation, estimate, equal, method, inverse		place value, multiply, multiplication, times, product, divide, division, factor, factor pairs, multiplication & division facts, operation, estimate, multiple, shared equally, array	

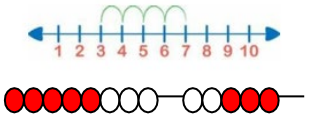
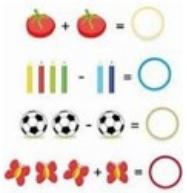
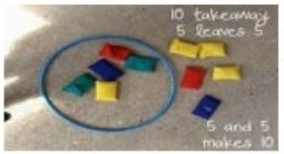
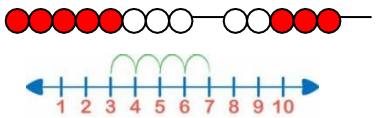
Upper Key Stage 2 - Year 5

	+	-	x	÷
National Curriculum Objective	Add and subtract whole numbers with more than 4 digits and decimals, using formal written methods of columnar addition and subtraction.		• Multiply numbers up to 4 digits by a 1 or 2-digit number. • Introduction of long multiplication with explanations. • Introduction of compact long multiplication.	Divide numbers up to 3 digits by a 1 digit number using the formal written method of short division and interpret remainders appropriately in context
Suggested calculation	Compact column		• Formal long multiplication	• Compact short division
Mathematical vocabulary	addition, subtraction, sum, total, difference, minus, less, column addition, column subtraction, operation, regroup, inverse, estimate, digit, place holder, rounding, approximate, accuracy		multiply, multiplication, times, product, commutative, short multiplication, long multiplication, multiplication facts, estimate, multiple, remainder	

Year 6

	+	-	x	÷
National Curriculum Objective	Add and subtract whole numbers with more than 4 digits, and decimals with different place values, using formal written methods of columnar addition and subtraction.		• Multiply multi-digit numbers up to 4 digits by a 2 digit whole number using formal written method of long multiplication.	Divide numbers up to 4 digits by a 2 digit number whole number using the formal written method of long division, and interpret remainders as whole number, fractions or decimals
Suggested calculation	Compact column		• Formal long multiplication	• Long Division
Mathematical vocabulary	addition, subtraction, sum, total, difference, minus, less, column, operation, inverse, estimate, approximate, multiply, multiplication, times, product, commutative, short multiplication, long multiplication, estimate, remainder, fraction, decimal, divisible			

Calculation Policy EYFS

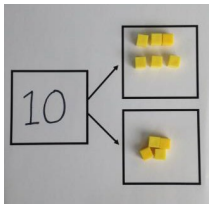
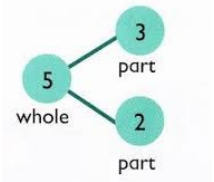
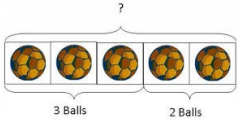
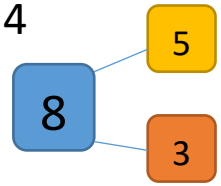
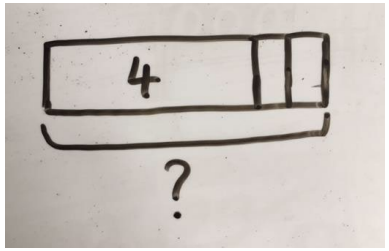
Addition	Subtraction	Multiplication	Division
Children are encouraged to gain a sense of the number system through the use of counting concrete objects.   Combine objects in practical ways and count all.  Understand addition as countingon and will count on inones and twos using objects,  cubes, bead string and number line. Use concrete and pictorial representation to record their calculations. Begin to use + and = Encourage to develop a mental picture of the number system in their heads to use for calculations.  Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.	Children are encouraged to gain a sense of the number system through the use of counting concrete objects.   Understand subtraction as counting out.  Begin to count back in ones and twos using objects, cubes, bead stringand number line.  They use concrete and pictorial representation to record their calculations. They begin to use - and = They are encouraged to develop a mental picture of the number system in their heads to use for calculations. Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.	Children use concrete objects to make and count equal groups of objects.  They will count on in twos using a beadstring and number line. They understand doubling as repeated addition. 2 + 2 = 4 They use concreteand pictorial representation to record their calculations.  Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.	Children use concrete objects to count and share equally into 2 groups. 6 cakes shared between 2 people each person gets 3 cakes. 6 ÷2 = 3   Count a set of objects and halvethem by making two equal groups. Understand sharing and halving as dividing by 2. Begin to use objects to makegroups of 2 from a given amount. Use concrete and pictorial representation to record their calculations.  Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.

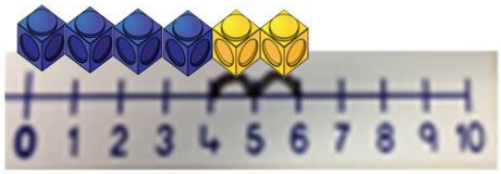
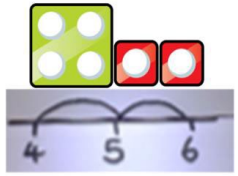
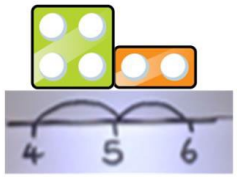
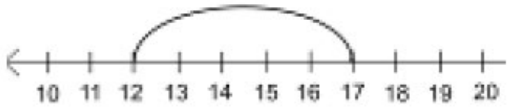
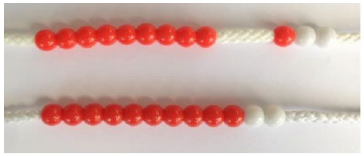
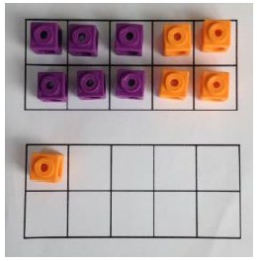
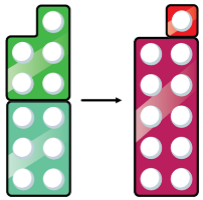
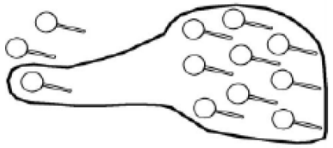
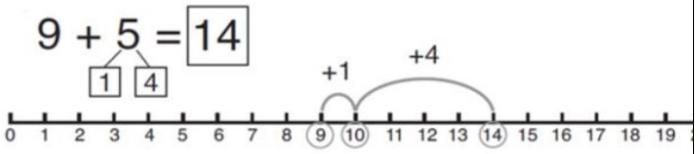
Progression in Written Calculation Years 1 - 6

Addition

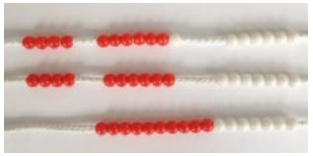
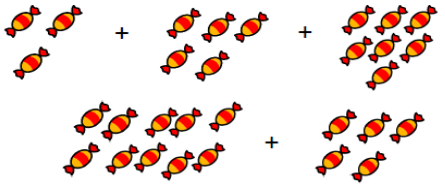
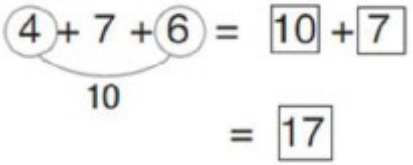
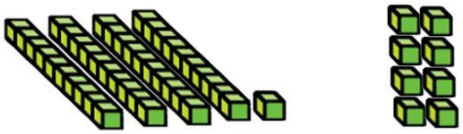
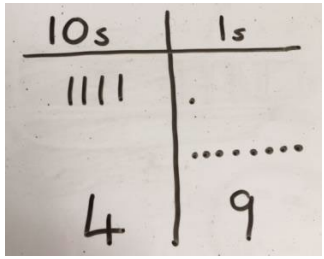
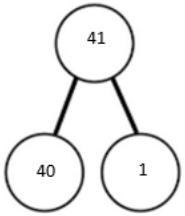
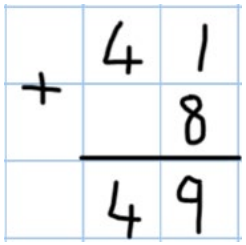
Addition and Subtraction are connected. Addition names the whole in terms of parts, while subtraction names a missing part of the whole.

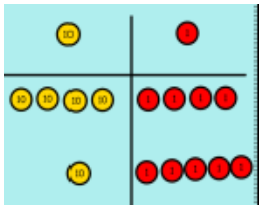
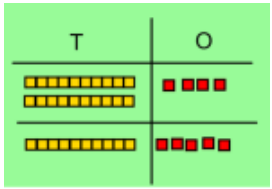
Part	Part
Whole	

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. Use cubes or Numicon	A bar model which encourages children to count on rather than count all 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 =

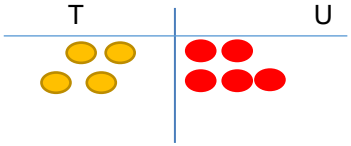
	  	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer. 12 + 5 = 17
Regrouping to make 10.	Using counters/cubes or Numicon  6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. 	Use pictures or a number line. Regroup or partition the smaller number to make 10.  3 + 9 = 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now? Develop an understanding of equality 6 + ☐ = 11 6 + 5 = 5 + ☐
Children should be here by the end of year 1			

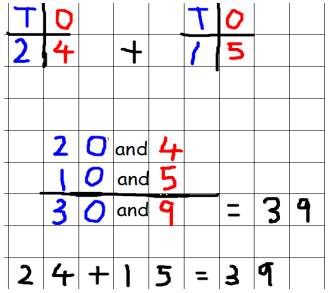
Year 2

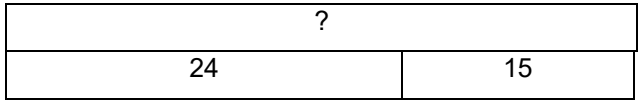
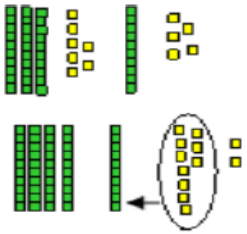
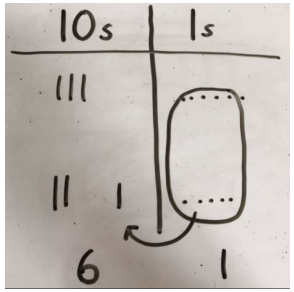
Adding three single digits	Use bead strings to work out sums e.g. 4 + 7 + 6 = 17. Put 4 and 6 together to make 10. Add on 7.  Build a tower of bricks and then ask the child to split them in 3 ways – add the numbers together.	Add together three groups of objects. Draw a picture to recombine the groups to make 10.  or Bar model represent the 3 numbers. ? (10, 20, 30)	Combine the two numbers to make 10 and then add on the remainder.  Children should be shown missing number sentences.
TensOnes + Ones	Continue to develop understanding of place value and partitioning e.g. 41 + 8 Using dienes or Cuisenaire rods to show bar models. 	Represent base 10 with lines / dots e.g. 41 + 8  The bar model: ? (8, 41)	41 + 8 1 + 8 = 9 40 + 9 = 49  

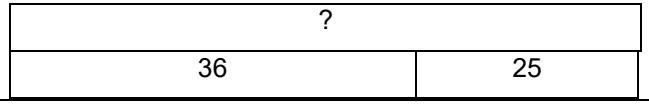
<table>
<tr>
<td>TO+ TO – no regrouping</td>
<td>Add together the ones first then add the tens.
Use the Base 10 blocks first before moving onto place value counters.
E.g. 24 + 15
 
 

<u>Partitioning (Aggregation model)</u>
34 + 23 = 57
<u>Base 10 equipment:</u>
 

Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.
Using dienes/Cuisenaire to show bar models.</td>
<td>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.
 

To explain:
 

The bar model:
 
</td>
<td><u>Calculations</u>
21 + 42 =
21
+ 42</td>
</tr>
<tr>
<td>TO + TO
(With regrouping in the ones)</td>
<td>Show regrouping using dienes: 36 + 25
37 + 15 = 52
 
Continue to develop understanding of partitioning and place value</td>
<td>Children to represent base 10 by drawing it in a place value chart.
 

The bar model:
 
</td>
<td>Looking for ways to make 10
36 + 25=
30 + 20 = 50
6 + 5 = 11
50 + 11 = 61
Formal method:
36
+25
61
1</td>
</tr>
</table>

Children should be here by the end of Y2 ?

Year 3

HTO + O
(No regrouping)

HTO + O
(With regrouping)

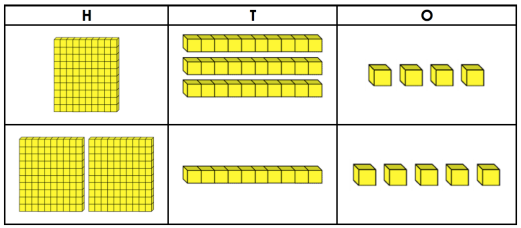
HTO + T
(No regrouping)

HTO + TO
(With regrouping in the tens)

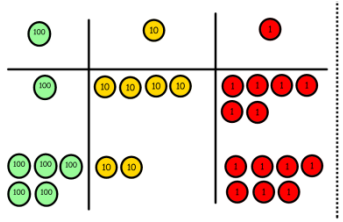
HTO + HTO
(With regrouping in ones & tens)

This can also be done with place value counters or Base 10.

134 + 215 =

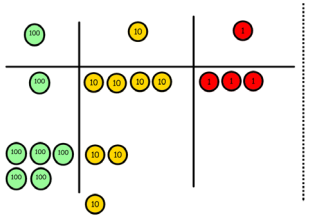


Make both numbers on a place value grid.



146
+ 527

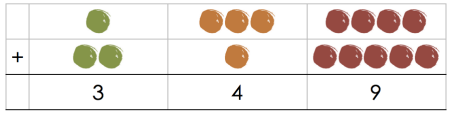
Add up the units and exchange 10 ones for one 10.



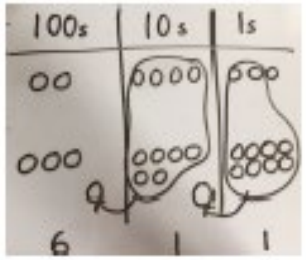
146
+ 527

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

Pictorial representation of the columns and counters.



:



If there are more than 10 or more counters in a column regroup into a new counter in the next column. Then add up all the columns. Make it, Draw it, Write it.

Bar models

?	
360	25

Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

Introduce the **column method.**

Regrouping:

compact column method up to 3 digits

$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline {\scriptstyle 1\ 1} \end{array}$$

	This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.		

Children should be here by the end of Y3

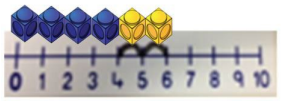
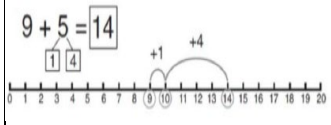
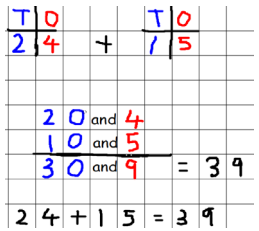
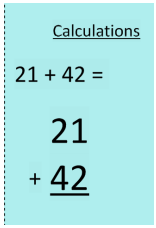
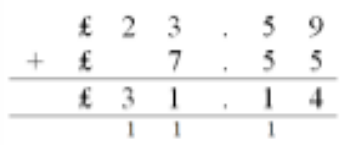
Y4 – **compact column method** up to 4 digits and two decimals (introduced with money) with the same number of digits. If there are more than 10 or more counters in a column regroup into a new counter in the next column. Then add up all the columns.

$$\begin{array}{r} 367 \\ +\ 85 \\ \hline 452 \\ 11 \end{array}$$

$$\begin{array}{rrrrrr} & £ & 2 & 3 & . & 5 & 9 \\ + & £ & & 7 & . & 5 & 5 \\ \hline & £ & 3 & 1 & . & 1 & 4 \\ \hline & & 1 & 1 & & 1 & \end{array}$$

Y5/Y6 – **compact column method** with more than four digits and decimals with different place value and regrouping in some columns.
Could use place value counters for adding decimals

$$\begin{array}{rrrrrr} 2 & 3 & . & 3 & 6 & 1 \\ & 9 & . & 0 & 8 & 0 \\ 5 & 9 & . & 7 & 7 & 0 \\ + & 1 & . & 3 & 0 & 0 \\ \hline 9 & 3 & . & 5 & 1 & 1 \\ \hline 2 & 1 & & 2 & & \end{array}$$

Quick Glance Addition Written Methods:		
Year Group	**Written Method**	**Written Method Example**
EYFS	Number tracks and Number lines	
Year 1	Number lines Understanding equality	 6 + ? = 11 6 + 5 = 5 + ? 6 + 5 = ? + 4
Year 2	Partitioning column compact column	 
Year 3	Expanded column Formal column	243 +368 611 1 1
Year 4	Formal column	Real life, money, measures. More than 2 numbers added 
Year 5	Formal column	Emphasis on decimals, money, measures, reasoning and worded problems
Year 6	Formal column	Reinforcing and securing all of the above

Quick Glance Addition Number Size	
Year Group	**Number size**
EYFS	Up to 2 digit + 1 digit
Year 1	Up to 2 digits + 2 digit
Year 2	Up to 2 digits + 2 digits
Year 3	Up to 3 digits (1000)
Year 4	Up to 4 digits including two decimal places
Year 5/6	More than 4 digits and decimals

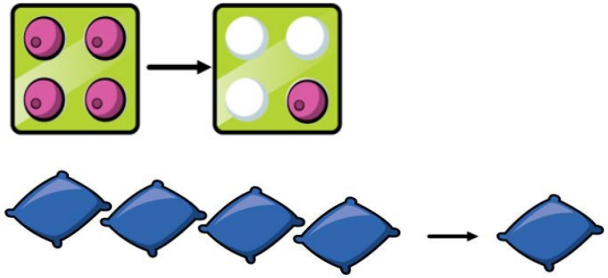
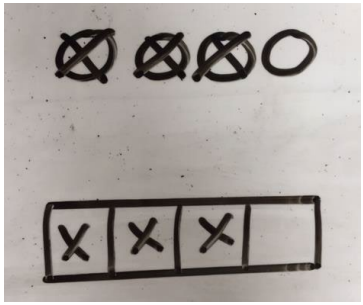
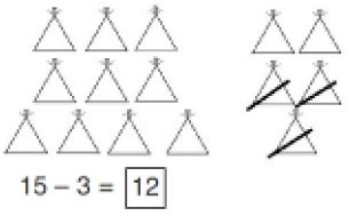
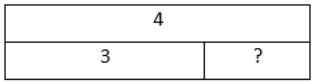
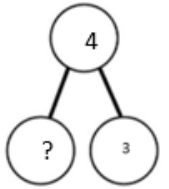
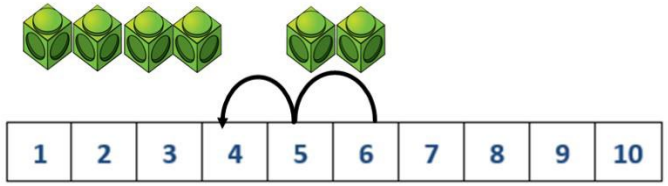
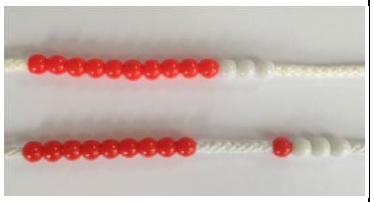
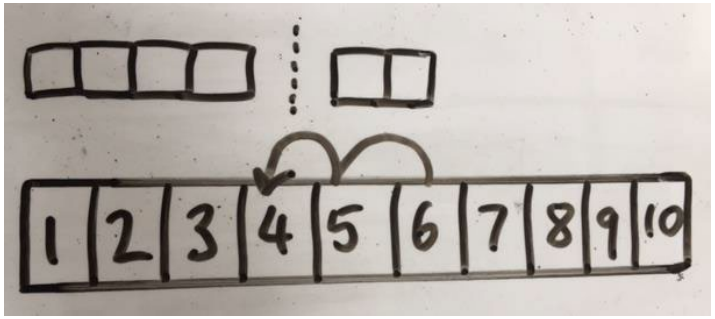
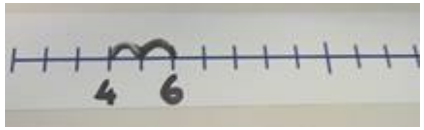
To add successfully, children need to be able to:

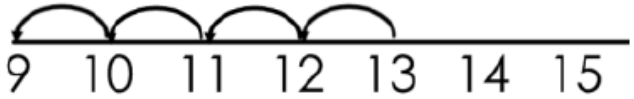
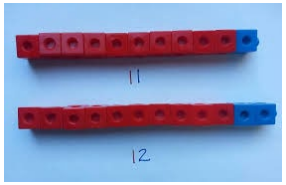
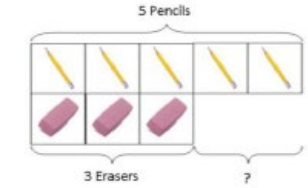
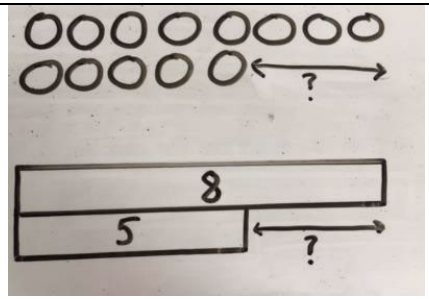
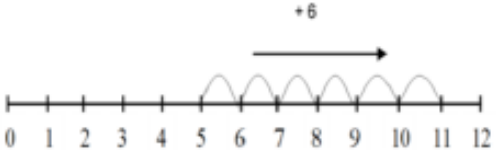
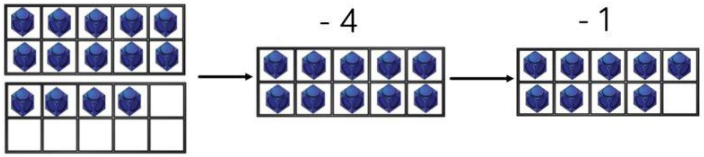
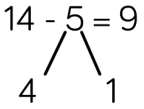
- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

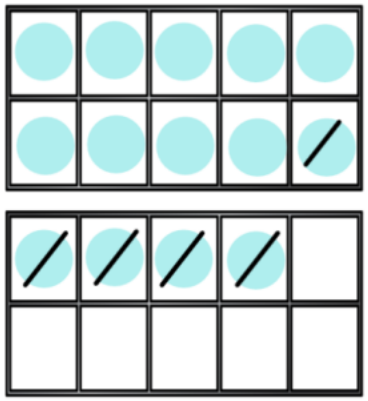
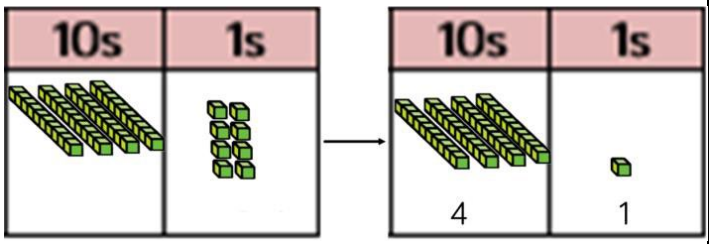
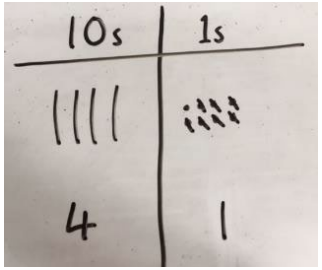
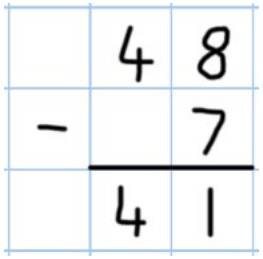
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

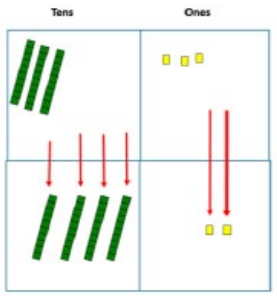
In Y3 and Y4 (and beyond where appropriate) the children should record their written calculations using H T O to reinforce the place value of each digit in the sum.

Subtraction

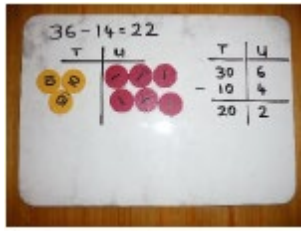
Objective and strategies	Concrete	Pictorial	
Taking away ones	Use physical objects, counters, cubes, Numiconand other items such as bean bags etc to show how objects can be taken away. 4 - 3 = 1 	Cross out drawn objects to show what has been taken away. Bar model can also be used.  	Children start to show recognisable abstract number sentences. 4 – 3 = 1 □ = 4 - 3  
Counting back	Use number lines or number tracks – start with 6 and count back 2 6 - 2 = 4  Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 	Children represent what they see pictorially eg. 	Children to represent the calculation on a number-line or number track and show their jumps. Encourage using an empty number-line.   Children start to show recognisable abstract number sentences.

	13 – 4	Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	6 – 2 = 4 Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference 	Draw cubes/other concrete objects or use the bar model to illustrate what they need to calculate  Count on to find the difference 	Find the difference between 8 and 5 8 – 5 = Explore why 9 – 6 = 8 – 5 Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Make 10	14 - 5 (Numicon, counters, 10 square, bead string)  Make 14 0n the ten frame Take away the 4 first to leave 10 Then takeaway 1 so you have taken away 5. You are left with the answer of 9.	Children present the ten frame pictorially and discuss what they did to make 10.	Show how to make 10 by partitioning  14 – 4 = 10 10 – 1 = 9 How many do we take off to reach the next 10? How many do we have left to take off?

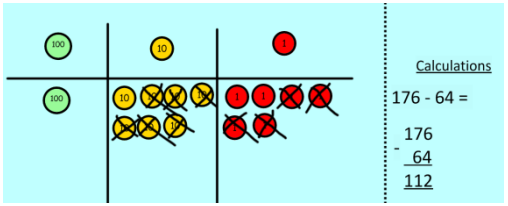
			Children start to show recognisable abstract number sentences. 13 - 7 = 6, 13 – 6 = 7
Children should be here by the end of Year 1			
Year 2			
TO – O (No exchanging)	Create the bigger number using dienes/place value counters and then subtract the smaller number. 48 – 7  Children should be advised to use mental methods to calculate this sum initially, before proving their answer with written methods.	Draw the dienes/place value counters and then cross out what you are subtracting.  The bar model: 48 ? | 7	Count back 7 or use column method 48 – 7 = 

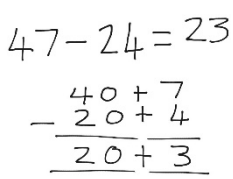
<table>
<tr>
<td>TO - TO Column method (without exchanging)</td>
<td>


Use Base 10 to make the bigger number then take the smaller number away.

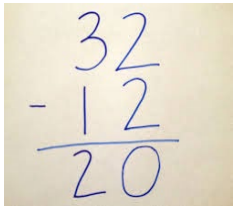
Show how you partition numbers to subtract. Again make the larger number first.


</td>
<td>

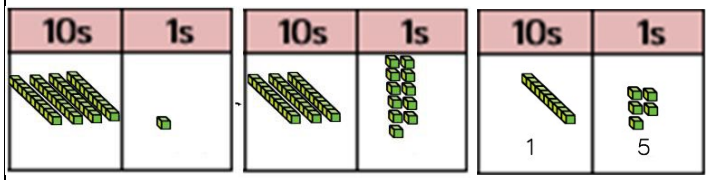
Draw the Base 10 or place value counters alongside the written calculation to help to show working.
</td>
<td>


This will lead to a clear written column subtraction.

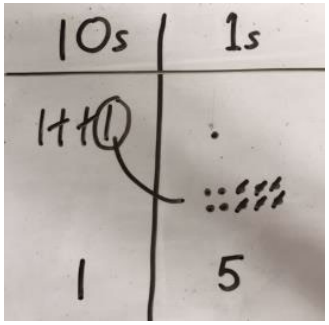


</td>
</tr>
<tr>
<td>TO - TO Column method with exchanging</td>
<td>Create the bigger number using dienes

41 - 26

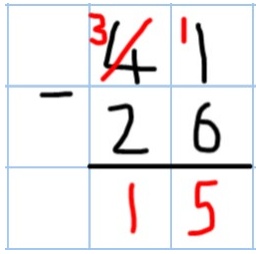


</td>
<td>Draw the dienesand then cross out what you are subtracting. The exchanging must be clearly shown. 41 - 26



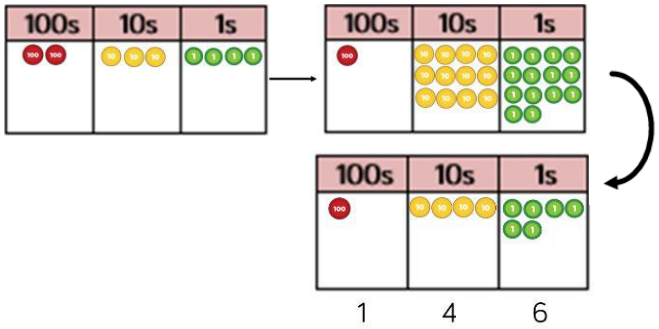
</td>
<td>Introduction of the column method:

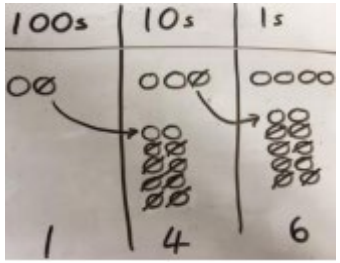
Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.



</td>
</tr>
<tr>
<td colspan="4">Children should be here by the end of Year 2</td>
</tr>
<tr>
<td colspan="4">Year 3</td>
</tr>
</table>

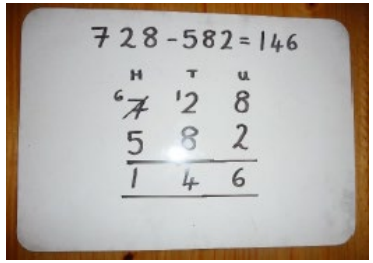
HTO - O (No exchanging)	Use dienes to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. 234-88	Represent the place value counters pictorially; remembering to show what has been exchanged. 234-88	**Formal column method** with exchanging. Children must understand what has happened when they have crossed out the digits.
HTO - O (With exchanging)			
HTO - TO (No exchanging)			
HTO - TO (With exchanging in the tens)			
HTO - TO (With exchanging in ones & tens)			





The bar model:

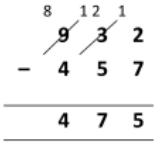
234	
?	88



This will lead to an understanding of subtracting any number including decimals.

Children should be here by the end of Year 3

Y4 – compact column method with regrouping with more than 3 digits. Include 0s. Make it, Draw it, Write it. Subtraction with money (decimals).



Answer: 475

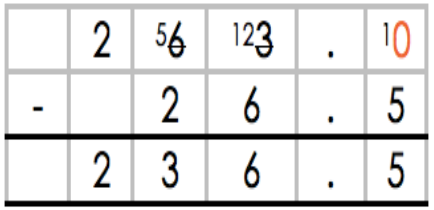
470 − 142

700 − 485

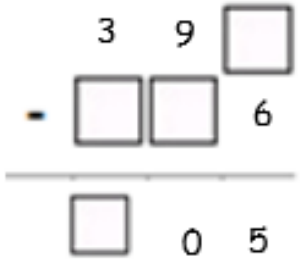
604 − 347

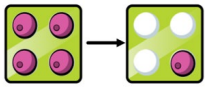
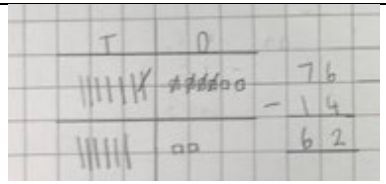
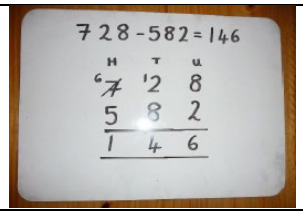
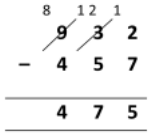
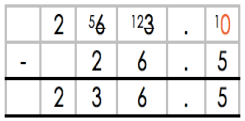
Y5 + Y6 compact column method with regrouping with more than 4 digits. . Place value counters for decimals with different amount of decimal places.

263 – 26.5 =



Missing digits:



Quick Glance Subtraction Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS	Taking away ones and Number lines	 4 - 3 = 1
Year 1	Number lines and Informal Partitioning	Children start to show recognisable abstract number sentences. 13 - 7 = 6, 13 – 6 = 7
Year 2	Partitioning column	
Year 3	Exchanging Formal column	
Year 4	Noughts Formal column	Real life, money, measures, 700 - 487  Answer: 475
Year 5	Formal column	Emphasis on decimals, money, measures, reasoning and worded problems 
Year 6	Formal column	Reinforcing and securing all of the above

Quick Glance Subtraction Number Size	
Year Group	**Number size**
EYFS	Up to 1 digit - 1 digit
Year 1	Up to 2 digits - 1 digit
Year 2	Up to 2 digits - 2 digits
Year 3	Up to 3 digits (1000)
Year 4	Up to 4 digits including two decimal places
Year 5/6	More than 4 digits and decimals

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

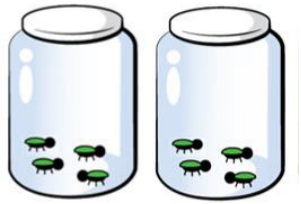
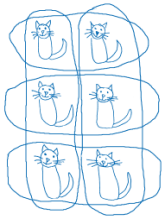
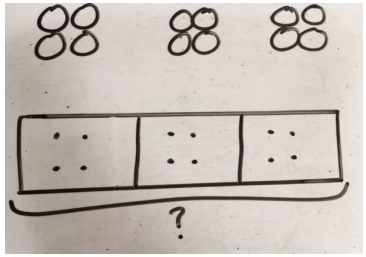
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

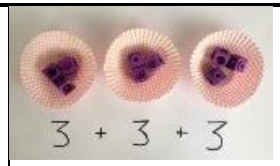
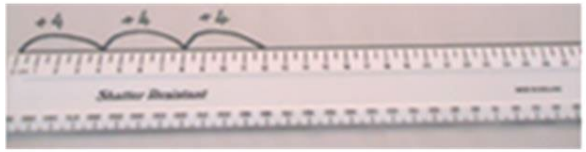
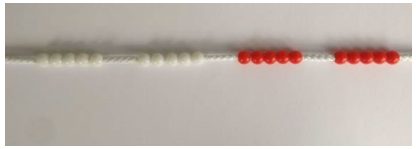
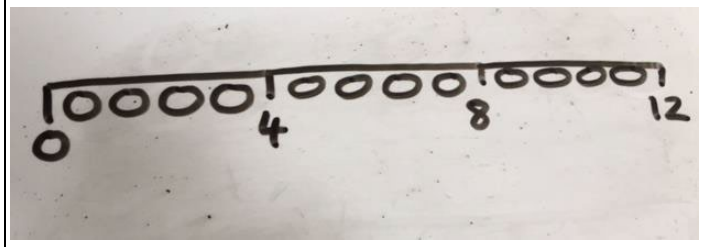
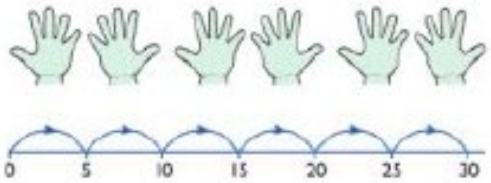
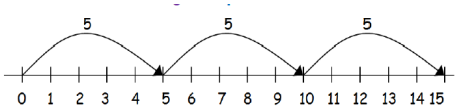
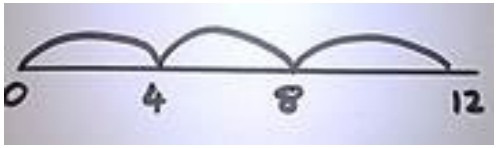
In Y3 and Y4 (and beyond where appropriate) the children should record their written calculations using H T O to reinforce the place value of each digit in the sum.

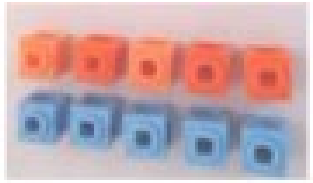
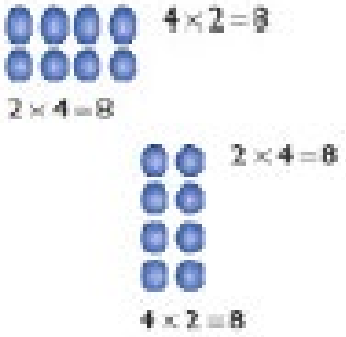
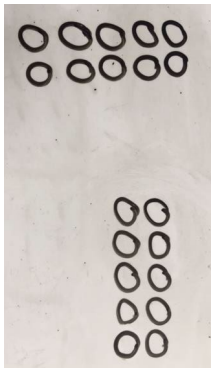
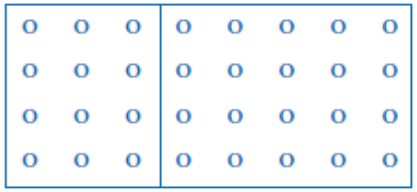
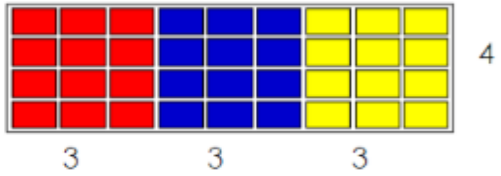
Multiplication

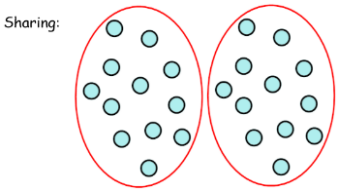
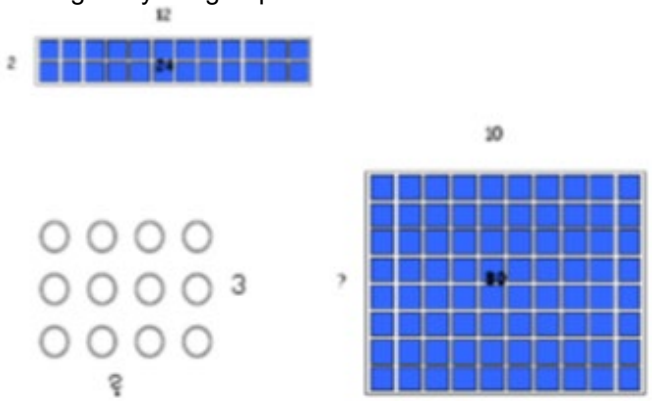
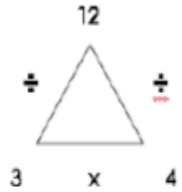
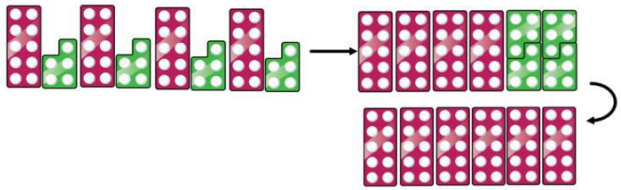
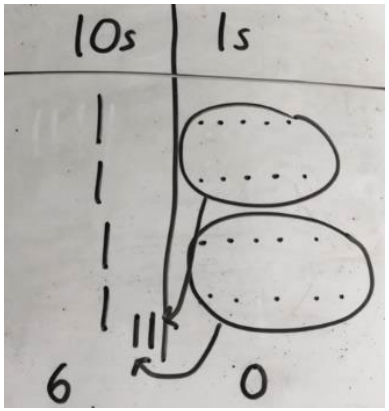
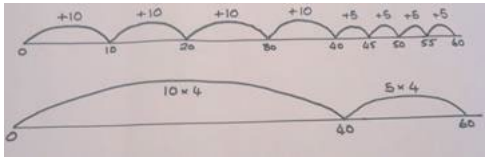
Multiplication and division are connected. Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			

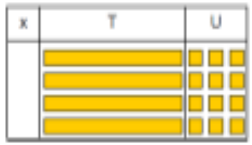
Objectives and strategies	Concrete	Pictorial	Abstract
Doubling / repeated grouping	Use a set of objects. Double the set by finding the same number again.  4 + 4 double 4 is 8 4 x 2 = 8  3 x 4 4 + 4 + 4 There are 3 equal groups with 4 in each group.	Draw the objects and use bar models  showing: 2 x 3 and 3 x 2  3 x 4 and 4 x 3	Children may start to show recognisable abstract number sentences. 3 x 4 = 12 4 + 4 + 4 = 12

Counting in multiples/ repeated addition	 There are 3 equal groups with 3 in each group. 3 x 4 (3 lots of 4) 4 x 3 (4, 3 times)   Use a bead string to show repeated addition.  Children use Cuisenaire Rods to partition totals into equal trains.  Using Numicon to show 3 x 5: 	Represent on the number line.Represent pictorially alongside a number line eg .    5 + 5 + 5 = 15	Abstract number line showing three jumps of four.  Children count in multiples of a number out loud. Write sequences with multiples of numbers. 2, 4, 6, 8 etc  2 + 2 + 2 + 2 + 2 = 10 Children are taught about the multiplication 'x' symbol. 3 x 4 = 12 is the same as 4 + 4 + 4 = 12

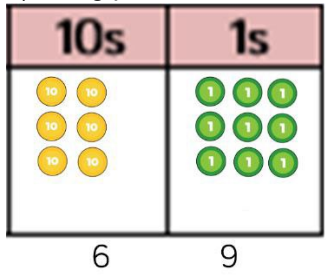
Introduction of using arrays to count in multiples of 2, 5, 10 (communtative law)	Use a set of objects. Children can place them in groups or start to focus them in on array shapes. 2 x 6 6 x 2  	Draw the objects in arrays. Draw in different rotations to find the communtative sentences. This prepares children for finding factors. Also, to help find the area of rectangles.  	Children start to use an array to write a range of abstract calculations. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 =10 10 = 5 + 5
Children should be here by the end of Year 1			
Year 2			
Consolidating use of arrays and repeated addition (distributive law)	32 pegs on a board are to be arranged into fours. How can these be shown? This shows the distributive law where 8 x 4 = 3 x 4 + 5 x 4. 	Ch to illustrate this in different ways and should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks. 	Which could also be seen as 9 x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 + 12 = 36 Or 3 x (3 x 4) = 36

Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 2 x ? = 24 	Drawing arrays or groups: 3 X ? = 12 	Introducing the Inverse operations Trios can be used to model the 4 related multiplication and division facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3  Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
Children should be here by the end of Year 2			
Year 3			
Partition to multiply	Use Numicon to show 15 x 4  Show multiplication of the 10s and the 1s separately	Children represent the concrete manipulative in a picture 	Children be encouraged to show the steps they have taken: 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used: 

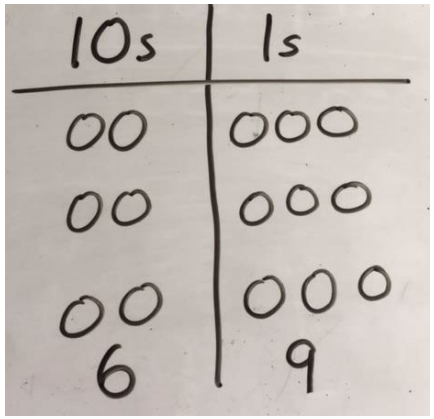
<table>
<tr>
<td>Formal column method
TO x O
(No regrouping)</td>
<td>1) Show the link with arrays with unifix 13 x 4

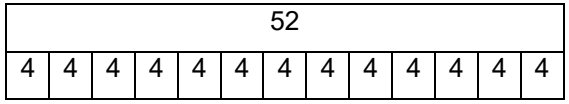
2) Using Dienes in a grid
 

4 rows of 13

3) Using place value counters
 

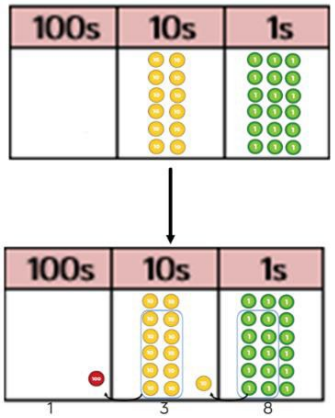
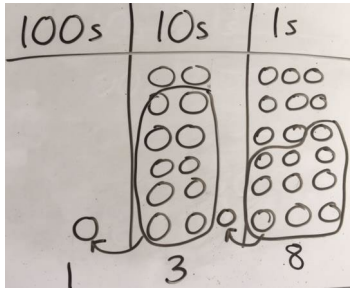
3 rows of 23

3 x 23</td>
<td>Children can represent the work they have done in a way that they understand.
 

Show in a bar model.
 
</td>
<td>Introduction with expanded short multiplication:
Children to record what it is they are doing to show understanding.

23
x3
9 (3x3)
+ 60 (20x3)
69 (23x3)

23
× 3
69</td>
</tr>
<tr>
<td>TO x O
(with regrouping of ones into tens)</td>
<td>Formal column method with place value counters.

6 x 23
 
</td>
<td>Children to represent the counters/base 10, pictorially e.g. the image below.
 
</td>
<td>Discuss how multiplying 6 x 2 gives you 12 and the answer is ten times bigger.

23
X 6
138
1 1</td>
</tr>
</table>

Children should be here by the end of Y3

Year 4

HTO x O (no regrouping)

HTO X O (extra digit in the answer)

HTO X O (with regrouping of ones into tens)

HTO X O (with regrouping of tens into hundreds)

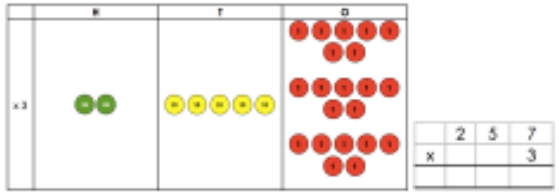
HTO X O (with regrouping of ones into tens and tens into hundreds)

Children can continue to be supported by place value counters at the stage of multiplication.

257 x 3 =
Use the place value counters to demonstrate multiplying in columns.

Make the number with the place value counters.

Start with the ones. Make 3 groups of 7. E.g. 3 x 7 = 21



If there are 10 or more counters in a column exchange for counters in the next highest column.
E.g. 20 ones for 2 tens

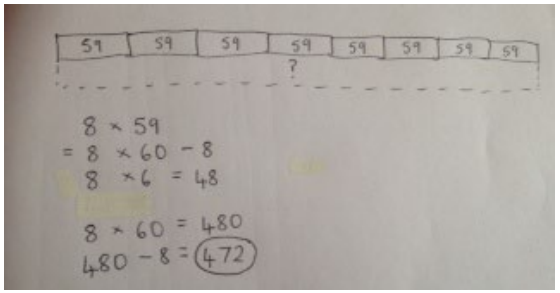
Children to represent the counters / dienes pictorially:

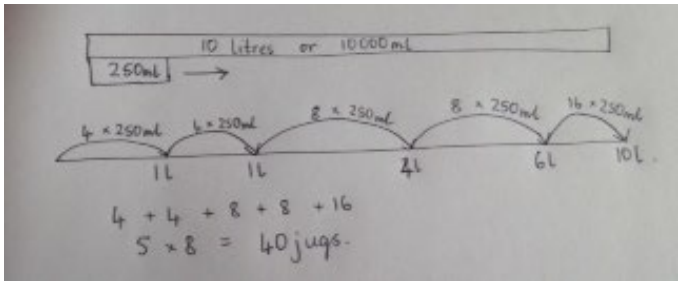
With regrouping of 1s into Tens, using dienes:

Show in a bar model.

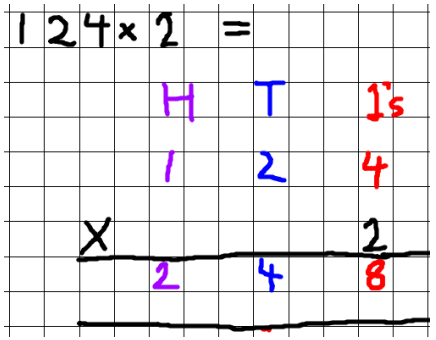
372		
124	124	124

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



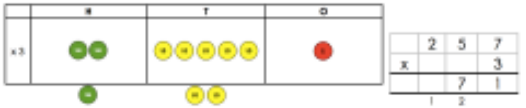


Introduction of **formal short multiplication:**

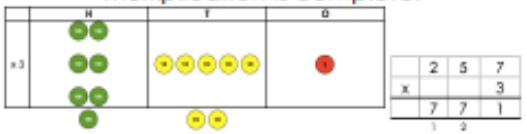


Now look at the tens, children make 3 groups of 50. E.g. 3 x 50 = 250

If there are 10 or more counters in a column exchange for the next highest column.
E.g. 10 tens for 1 hundred
The previously exchanged counters are added to the product.



Now look at the hundreds, make 3 groups of 200. E.g. 200 x 3 = 600
The previously exchanged counters are added to the product and the multiplication is complete.

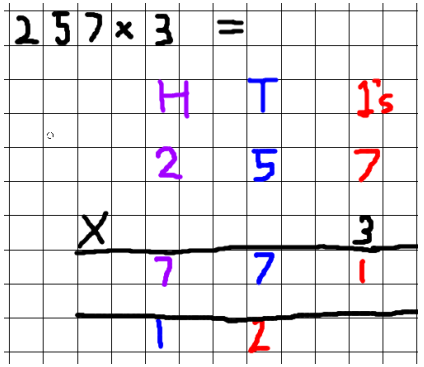


It is important at this stage that they always multiply the ones first and note down their answer followed by the tens, then hundreds etc.

Show in a bar model.

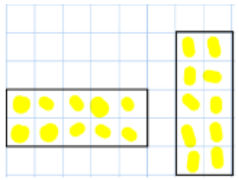
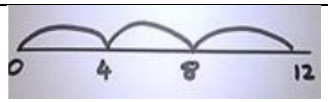
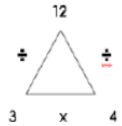
771		
257	257	257

With regrouping



Children should be here by the end of Y4

Y5 - Introduction of **long multiplication** with up to 4 x 2 digits. reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. **3 2** **X 2 4** **1 2 8 (4 x 32)** **+ 6 4 0 (20 x 32)** **7 6 8 (24 x 32)** Onto more compact methods	**Y5** - Introduction of **compact long multiplication** with up to 4 x 2 digits**.** **Onto multiplication of decimals.** To get 744 children have solved 6 × 124. To get 2480 they have solved 20 × 124. 124 x 26 becomes **1 2 4** **X 2 6** **7 4 4** **+ 2 4 8 0** **3 2 2 4** 1 1 **1 2 4** **X 2 . 6** **7 4 . 4 (0.6 x 124)** **+ 2 4 8 . 0 (2 x 124)** **3 2 2 . 4 (2.6 x 124)** When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract	**Y6** – Consolidation of **compact short multiplication** and **compact long multiplication** methods with up to 4 digits by a 2 digit. Also onto **multiplication of decimals by decimals** – estimation first, then adding no of decimal places. **1 . 2 4** **X 2 . 6** **7 4 4** **+ 2 4 8 0** **3 . 2 2 4** 1 1

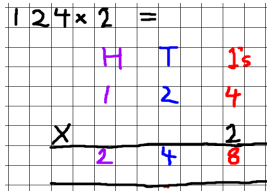
Quick Glance Multiplication Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS	Arrays	
Year 1	Arrays and repeated addition	
Year 2	Arrays in a grid Introducing inverse partitioning	3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3 
Year 3	Formal short multiplication	23 × 3 69

To multiply successfully, children need to be able to:

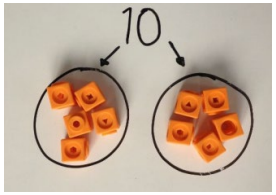
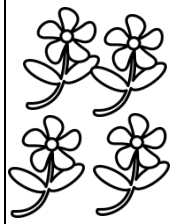
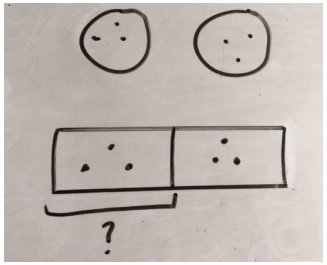
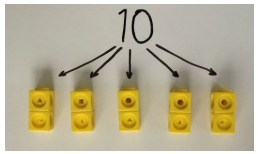
- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

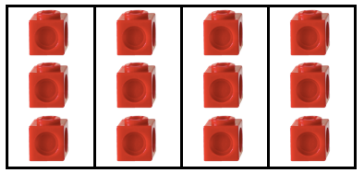
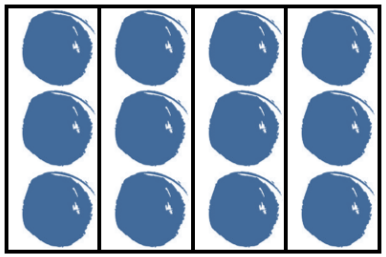
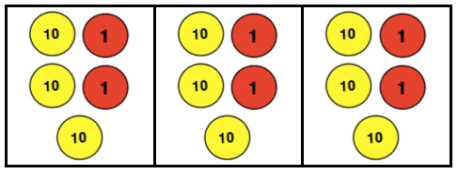
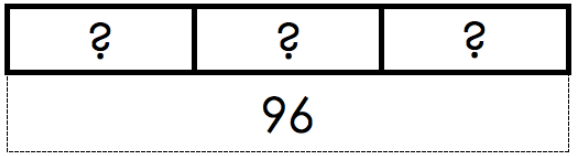
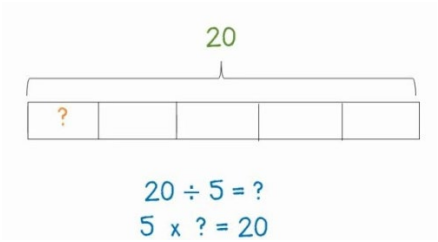
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

In Y3 and Y4 (and beyond where appropriate) the children should record their written calculations using Th H T O to reinforce the place value of each digit in the sum.

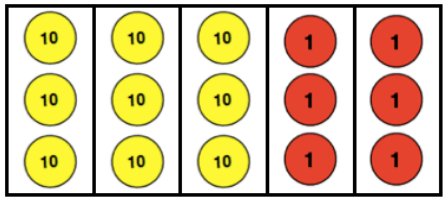
Year 4	Expanded short multiplication Formal short multiplication Multiplying by 10, 100, 1000	**Compact Short:** 
Year 5	Formal long multiplication including whole numbers by decimals Multiplying decimals by 10, 100, 1000	**Compact Long:** Emphasis on decimals, money, measures, reasoning and worded problems **3 2** **X 2 4** **1 2 8 (4 x 32)** **+ 6 4 0 (20 x 32)** **7 6 8 (24 x 32)** 124 × 26 becomes 1 2 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1
Year 6	Formal long multiplication Including onto multiplying decimals by decimals	**1.2 4** **X 2.6** **7 4 4** **+ 2 4 8 0** **3.2 2 4**

Division

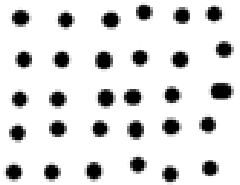
Objectives and strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups Share objects into groups. I have 12 cubes.  Can they be shared equally in 3 groups? After sharing between 3 groups we have found that are 4 in each group.  12 ÷ 3 = 4 ?	Children use pictures or shapes to share quantities.    8 ÷ 2 = 4  12 ÷ 3 = 4	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts. Share 9 buns between three people. 9 ÷ 3 = 3 Share 12 sweets between 3 people. 12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups.  	.	12 ÷ 3 = 4 Divide 12 into groups of 3 . How many are in each group?

	After making groups of 3 we discovered there were 4 of them.  12 ÷ 3 = 4	 12 ÷ 3 = 4	
Children should be here by the end of Y1			
Year 2			
Sharing objects into groups	Using place value counters e.g. 96 ÷ 3 = 32  After sharing we found there were 3 tens and 2 ones in each group.	Use pictures or shapes to share quantities. Bar Modelling: Split the bar into the number of groups you are dividing by and work out how many would be within each group.  96 ÷ 3 = Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group 	Share £96 between 3 children. £96 ÷ 3 = £32

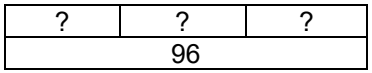
<table>
<tr>
<td>Grouping objects</td>
<td>Divide quantities into equal sized groups. Use cubes, counters, objects or place value counters to aid understanding.
I have 12 cubes.

Using place value counters e.g. 96 ÷ 3 = 32
 

After making groups of 3, we find there were 3 groups of ten and 2 groups of one.</td>
<td>Represent using arrays: How many strawberries will each child have if 30 are shared between 5 children?

30 ÷ 6 = 5
 
30 ÷ 5 = 6

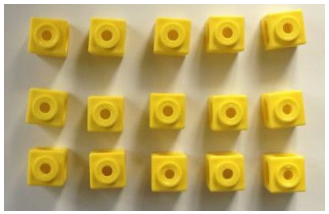
Bar Modelling:
You know how many would be within each group, but need to find out how many groups.

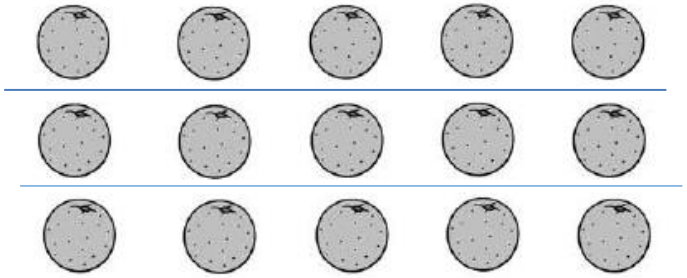
96 ÷ ? = 3</td>
<td>Sweets are sold in bags of 3. If I have 12 sweets how many bags would I need?

12 ÷ 3 = 4

There are 96 children sitting in rows of 3.
How many rows are there?

96 ÷ 3 = 32</td>
</tr>
<tr>
<td>Division within arrays</td>
<td> 
Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg 15 ÷ 3 = 5 5 x 3 = 15

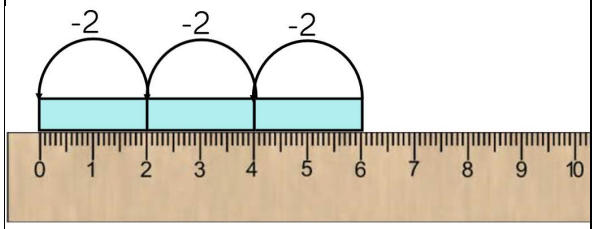
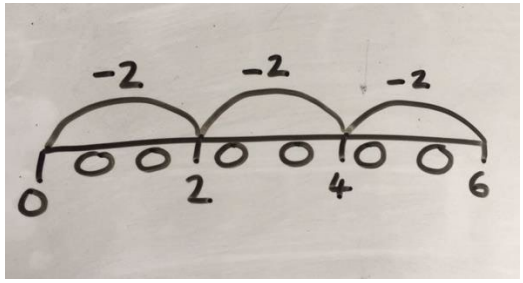
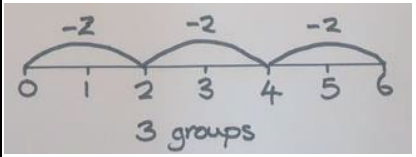
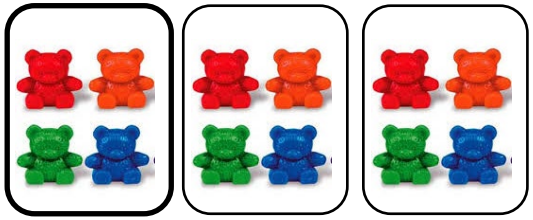
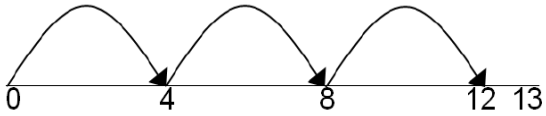
15 ÷ 5 = 3 3 x 5 = 15</td>
<td> 
Draw an array and use lines to split the array into groups to make multiplication and division sentences.</td>
<td>Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28

4 x 7 = 28

28 ÷ 7 = 4

28 ÷ 4 = 7</td>
</tr>
</table>

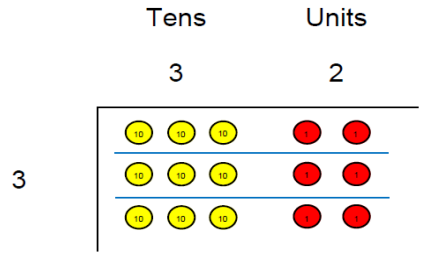
Repeated subtraction	Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted 
Children should be here by the end of Y2			
Year 3			
Division with a remainder	14 ÷ 3 = 4 r 2 Divide objects between groups and see how much is left over  	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  13 ÷ 4 = 3 r 1 Draw dots and group them to divide an amount and clearly show a remainder.  Bar model can be used to display whole being divided into equal parts with a remainder left over. 5 | 5 | 5 | R2 17	Complete written divisions and show the remainder using r.  14 ÷ 4 = 3 r2 17 ÷ 3 = 5 r 2

Short division

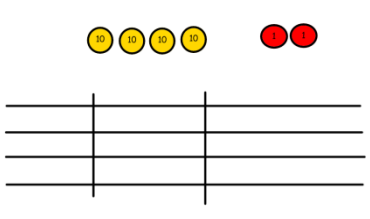
No remainder, Exchanging

Remainder, no carrying

HTO ÷ O
(No regrouping and no remainder)



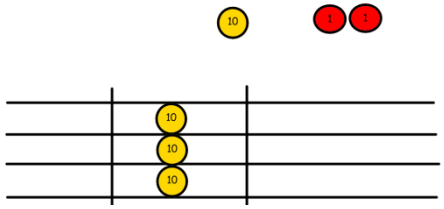
Use place value counters to divide using the bus stop method alongside



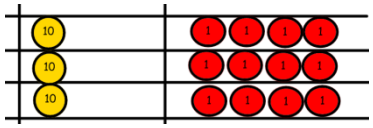
Calculations
42 ÷ 3

42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

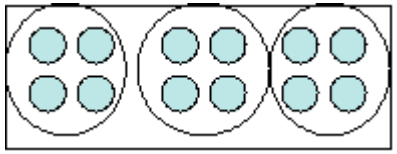


Exchange this ten for ten ones and then share the ones equally among the groups.

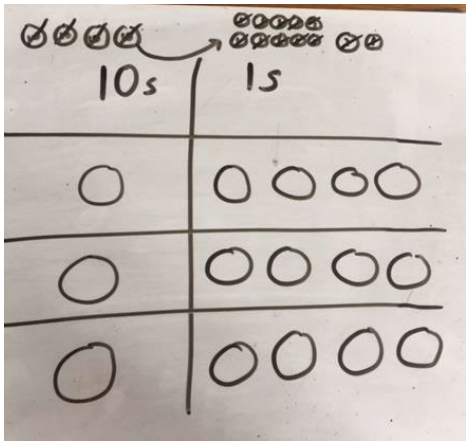


We look how much in 1 group so the answer is 14

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.



Children to be able to make sense of the place value counters and write calculations to show the process

.
42 ÷ 3
42 = 30 + 12
30 ÷ 3 = 10
12 ÷ 3 = 4
10 + 4 = 14

42 ÷ 3 = 14

$$\begin{array}{r} 232 \\ 3\overline{)696} \end{array}$$

Children should be here by the end of Y3

Year 4

Short division

HTO ÷ O (No regrouping and no remainder)

HTO ÷ O (No regrouping and a remainder)

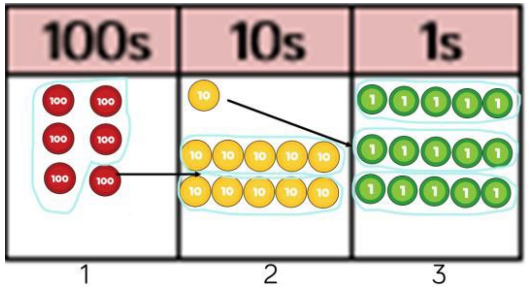
HTO ÷ O (Regrouping of hundreds into tens)

HTO ÷ O (Regrouping of tens into ones)

HTO ÷ O (Regrouping of hundreds into tens and tens into ones and a remainder

Where there are zeros in the quotient (e.g. 816 ÷ 4 = 204)

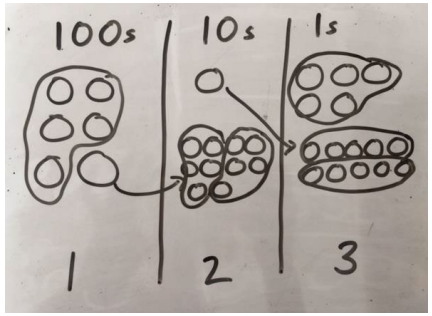
Short division using place value counters to group. 615 ÷ 5



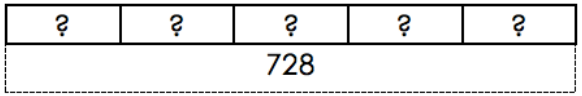
1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Draw the counters and cross out any that are regrouped.

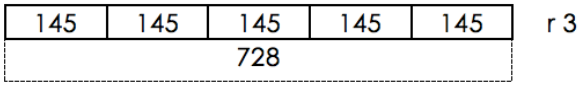
.



Bar modelling can support learners when solving problems with division alongside the formal written methods.



728 ÷ 5 = 145 r 3



Introduce **short division method.**

Begin with divisions that divide equally with no remainder

$$5\overline{)615} = 123$$

Move onto divisions with a remainder.

	1	4	5	r3
5	7	22	28	

$$5\overline{)432} = 86 \text{ r } 2$$

Children should be here by the end of Y4

Year 5

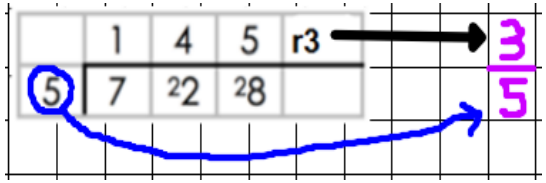
Short division

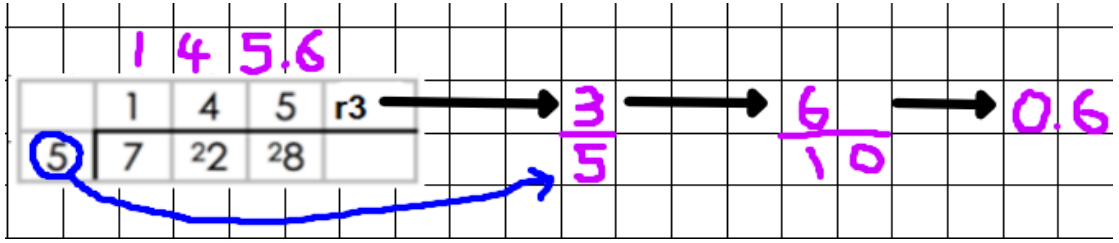
Remainder expressed as a fraction

Remainder expressed as a simplified fraction

Remainder expressed as a decimal

Revision of **short division method:**





total

Finally move into decimal places to divide the accurately.

```
        1 4 . 6
      ------------
3 5 | 5 1 1 . 0
      16   21
```

Children should be here by the end of Y5

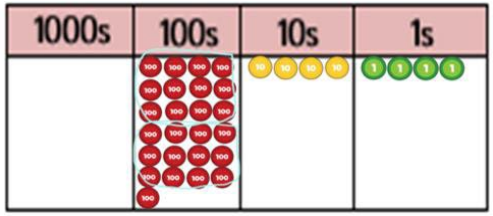
Year 6

Long division

(2 digit divisors)

Could introduce long division using place value counters 2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.

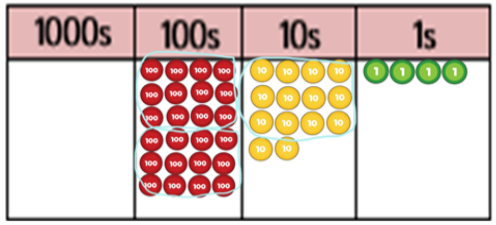


We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

```
      02
   ------
12 | 2544
     24
     --
      1
```

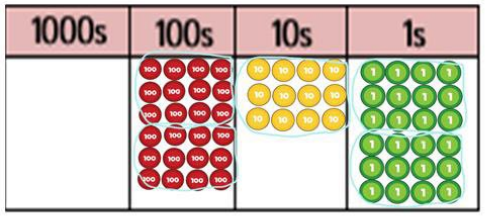
```
      0212
   ------
12 | 2544
     24
     --
der. 14
     12
     --
      24
      24
      --
       0
```

9382 ÷ 37



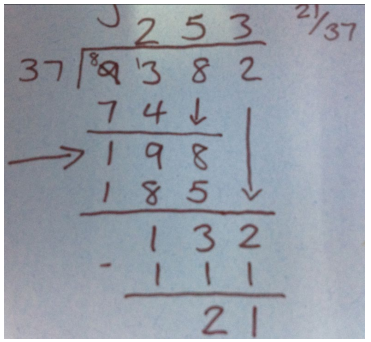
After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$



Quick Glance Division Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS **Year 1**	Sharing and grouping in circles	Sharing: 3 3 3 3 Grouping:
Year 2	Sharing and grouping in arrays	96 ÷ 3 = 32
Year 3	Sharing within place value columns Remainders	42 ÷ 3 = 14 17 ÷ 3 = 5 r 2

Year 4	Compact short division	**Compact short division:**
Year 5	Compact short division Remainders - or written as mixed number / decimal	145 r3 **or** 145 **2/5** **or** 145.6
Year 6	Long division	**Long division:**

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.
-

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.To carry out written methods of division successful, children also need to be able to:

- **estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;**
- **multiply a two-digit number by a single-digit number mentally;**
- **subtract numbers using the column method.**

Quick Glance Division Number Size (Children must stay within these boundaries)	
Year Group	**Number size**
EYFS/Year 1	Up to 2 digits ÷ 1 digit
Year 2	Up to 2 digits ÷ 1 digit
Year 3	Up to 2 digits ÷ 1 digit
Year 4	Up to 3 digits ÷ 1 digit
Year 5	Up to 3 digits ÷ 1 digit
Year 6	Up to 4 digits ÷ 2 digit

Reviewed by Michelle Tippett-Nestor	Autumn Term 2023		
Next Revision (Please highlight as appropriate)	Annual	Bi-annual	Tri- annual
To be reviewed	Autumn Term 2024		