'Shine like stars in the world.'
Philippians 2:15

St Paul's CE Primary School

Maths Written-Calculation Policy

Vision

Our school is one family, united in love and deeply rooted in our Christian values, where together on life's journey we flourish, striving for excellence in all that we do. Inspired by the transformation of St Paul, and enlightened by the glory of God, we will shine like stars to make the world a better place.

Values

Love, Forgiveness, Faith, Friendship, Hope and Peace.

Rationale

This policy has been designed to show progression in written mathematical methods throughout the school. Our written calculation policy is set out to show:

- The objectives stipulated for all four operations by the National Curriculum
- The calculation methods for each year group
- Relevant vocabulary needed at each stage
- Examples of reasoning activities

Concrete, Pictorial and Abstract (CPA)

Each calculation method is shown through **concrete, pictorial and abstract** examples.

- From EYFS to Year 6, new calculation methods are introduced using concrete manipulatives.
- Pupils then progress to pictorial representations before working abstractly. The bar model is used during the pictorial stage to support understanding.
- Progression through the stages varies according to individual pupil need.

Mastering Calculation

The curriculum places a strong emphasis on mastery. If a pupil is fluent in a calculation method for their year group, they will not just move on to a different method. Instead, pupils are encouraged to deepen their understanding within the same method. This may include:

- Using the method in different contexts
- Applying the method to other areas of learning
- Solving calculations with missing digits or values
- Explaining or exploring different aspects of the method
- Proving answers using pictorial representations or manipulatives
- Identifying and explaining errors in calculations

Mathematical Vocabulary

The National Curriculum places great emphasis upon the use of correct mathematical vocabulary and children developing this. Throughout school, children are strongly recommended to use and apply mathematical vocabulary when learning a new method or concept. They will be constantly exposed to this, have it expertly modelled by their teacher and be expected to use it themselves when justifying methods.

Mental Methods

Mental strategies are prioritised alongside written methods. Estimation and inverse checking are taught explicitly**.** Children will be shown number patterns and relationships between numbers throughout the school. Times tables are introduced and taught in specific year groups:

Reception: multiples of 2
Year 1: multiples of 2, 5 and 10
Year 2: 2x, x5, x10 (introduction to multiples of 3)
Year 3: 2x, 3x, x4, x5, x6, x8, x10
Year 4: (as above) including x7, x9, x11, x12 **MTC test**
Year 5 & 6: All tables up to 12 x 12

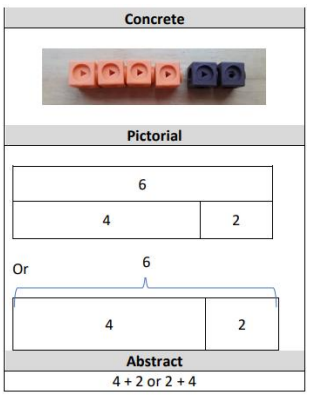
Bar Models

Bar models are used to:

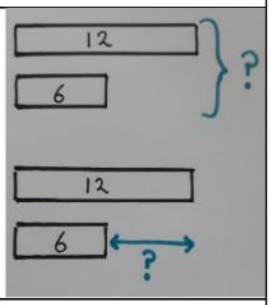
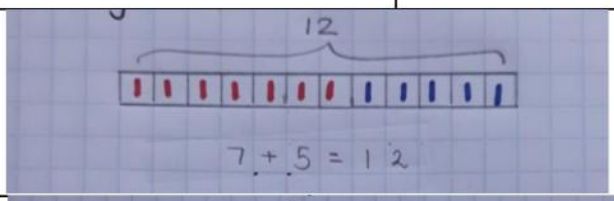
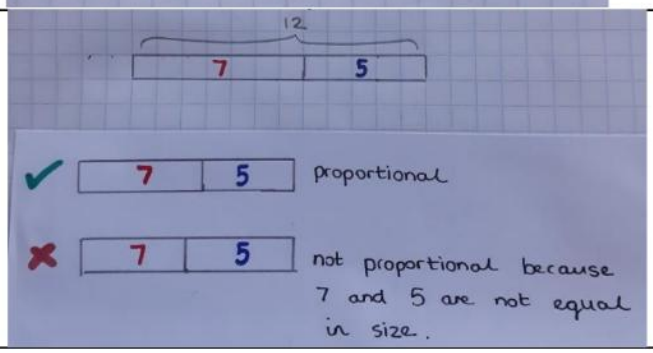
- represent underlying mathematical structures
- support visualisation of mathematical problems
- develop problem-solving and reasoning skills

Bar modelling:

- is introduced in EYFS, including associated structures and vocabulary
- uses concrete representations to support understanding
- is used consistently across the school



Different Types of Bar Models

Part/whole bar models	***Comparison bar models***
<table><tr><td colspan="2">whole</td></tr><tr><td>part</td><td>part</td></tr></table> The whole is the sum of the parts.	Comparing two amounts by drawing their bars and having brackets represent 'the whole' or 'the difference'. 
Discrete bar models Every unit is an individual box.	
Continuous bar models Amounts are represented as proportional rectangles.	

EYFS	Year 1	Year 2	KS2
• Concrete exploration • Present items in a linear fashion. • Look at and discuss bar models with pictures in e.g. 5s and 10s frames • Not expected to draw accurate models independently though could start drawing boxes around objects like a bar model • Children should not be discouraged if they try to draw bar model jottings.	• Draw discrete bar models accurately and independently. • Use brackets for the whole but be exposed to diagrams where the whole is represented as a bar • Look at and discuss continuous models. • Begin to use continuous models where it becomes inefficient to draw discrete models.	Make a transition from discrete to continuous for most areas of maths and be able to draw these independently and accurately with increasing levels of proportionality.	Use continuous models with increasing levels of proportionality and variation in where the whole is depicted.

Progression in vocabulary of bar models:

EYFS	Year 1	Year 2	KS2
• Children should understand and identify parts and wholes. • Not expected to call them bar models.	• Children use part and whole vocabulary • Children can identify them as bar models	• Children confidently use part and whole vocabulary • Brackets terminology used when comparing whole bar to brackets drawn previously in year 1	Children can explain all aspects of a bar model, including parts/wholes, known/unknown and brackets/bars

Mathematical Vocabulary Progression

The calculation policy includes a progression of vocabulary from **EYFS** to **Year 6**. This is informed by the **National Curriculum's** requirement that pupils should read, spell and pronounce mathematical vocabulary correctly and confidently (National Curriculum, DfE). Vocabulary is cumulative; the table shows new vocabulary introduced each year.

	EYFS Vocabulary is developed through talk, play and practical experiences and is revisited and extended throughout Key Stage 1	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Number and Place Value	amount, count, subitise, numeral, digit, zero, less, more, fewer, same, equal, after, before, different, order, forwards, backwards, number line, odd, even	place value, value, ones, tens, sort, part, whole, compare, count on, digit, fewest, greatest, less than, greater than, partition, represent, ordinal numbers, numbers to 100	count in steps, count in multiples, digit value, estimate, rounding, compare (formal), interval, least	ascending, descending, hundreds, round, rounding, numbers to 1000, Roman numerals (to 12), place holder	thousands, digit value (extended), negative numbers, numbers to 10,000 Roman numerals (to 100)	ten thousands, hundred thousands, millions, power of 10, integer, rounding (extended), numbers to 1,000,000 Roman numerals (to 1000)	ten millions, approximate, digit value (refined)

Addition and Subtraction	1 more, 1 less, add, add more, plus, altogether, first, take away, number bonds, part, whole, numbers from zero to 20 (and beyond), double, half	addition, count on, count back, total, number facts, fact family, subtraction, minus, difference, between, equals, equals sign, symbol, number sentence	partition, bridging (through 10), calculate, calculation, exchange, regroup, multiple of, increase, decrease, sum, crossing the 10, crossing the 100, related facts, operation	column method, column addition, column subtraction, efficient method, estimate, inverse	efficient	accurate, approximate, constant difference, strategy,	order of operations
Multiplication And division	double, share, equal groups, groups of, lots of, unequal, odd, even, grouping, sharing,	multiples, array, repeated addition	multiply, multiple, divide, times table, product, fact family, inverse operation	factor, equal groups (formal), times (formal), divided by, divisor, quotient, commutative, scaling	scaling, factor pairs, inverse, remainder	square number, prime number, cube number, common factor, long multiplication, short division, remainder (fraction), remainder (decimal)	composite number, powers of

EYFS

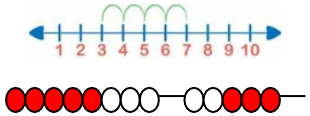
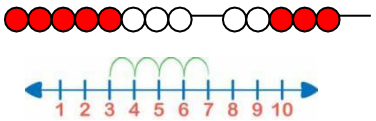
Children at the expected level of development will:

Number

- Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order;
- Subitise (recognise quantities without counting) up to 5;
- Automatically recall number bonds for numbers 0-5 and for 10, including corresponding partitioning facts.

Numerical Patterns

- Automatically recall double facts up to 5+5;
- Compare sets of objects up to 10 in different contexts, considering size and difference;
- Explore patterns of numbers within numbers up to 10, including evens and odds.

Addition	Subtraction	Multiplication	Division
Children are encouraged to gain a sense of the number system through the use of counting concrete objects.  Combine objects in practical ways and count all.  Understand addition as counting on and will count on in ones and twos using objects, cubes, bead string and number  	Children are encouraged to gain a sense of the number system through the use of counting concrete objects.  Understand subtraction as counting out.  Begin to count back in ones and twos using objects, cubes, bead string and number line.  	Children use concrete objects to make and count equal groups of objects.  They will count on in twos using a bead string and number line. They understand doubling as repeated addition. 2 + 2 = 4 They use concrete and pictorial representation to record their calculations. 	Children use concrete objects to count and share equally into 2 groups. 6 cakes shared between 2 people each person gets 3 cakes. 6 ÷2 = 3   Count a set of objects and halve them by making two equal groups. Understand sharing and halving as dividing by 2. Begin to use objects to make groups of 2 from a given amount.

line.	They use concrete and pictorial representation to record their calculations.	Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.	Use concrete and pictorial representation to record their calculations.  Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation

Bar Models- Addition

For all of the following areas, progression begins with the use of real life objects and moves to cubes/counters. The final stage would be for children to draw boxes around objects to show they are parts of a bar.

In EYFS, the 5s frame (or 10s frame) can be used to stimulate mathematical talk and exposure to a 'bar 'representing parts if the objects are placed in a linear fashion.



How many have we got? What is our whole? How many spaces are there? How many could we have?



What do

Bar Models-Subtraction



5 is our whole. 5 take away 1 is?



How many in each part? 5 is still our whole amount.

They begin to use - and =

They are encouraged to develop a mental picture of the number system in their heads to use for calculations. Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.

Bar Models - Doubling



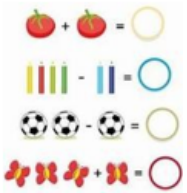
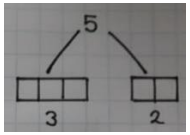
We doubled this part [the four]. How many do we have now? 8 is our whole.

Bar Models - Halving

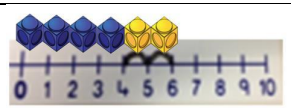
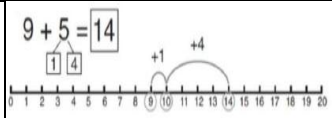
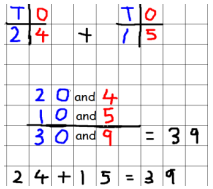
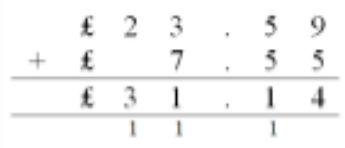


How many did we start with? 6 was our whole. We halved it [either splitting or sharing]. We have 2 parts now. Half of 6 is 3.

you notice? What's happened? Is this still 3? What is our whole?  Show me one more. • Show me one less. • How many do we have now? • What is our whole? • How many more can we have? Then how many would we have? What would our whole be? Use concrete and pictorial representation to record their calculations.  3 add 2 equals 5. 5 is our whole. We added these two parts together.			

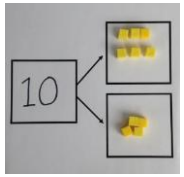
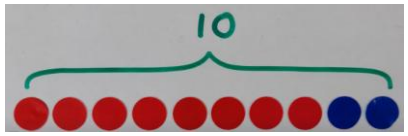
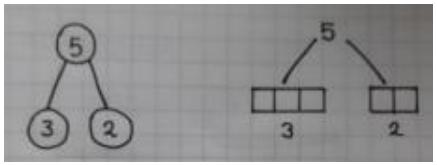
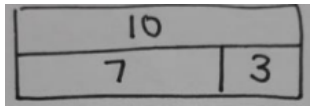
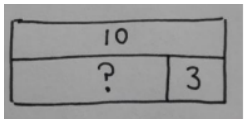
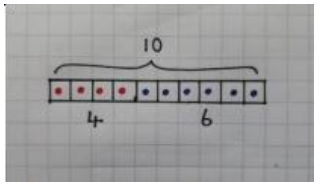
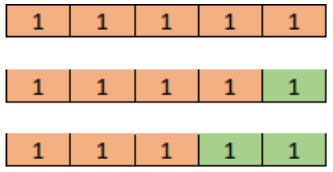
Begin to use + and = Encourage to develop a mental picture of the number system in their heads to use for calculations.  Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation. **Number Bonds** Using both of these representations for number bonds will ensure children are provided with variation in their representations and also begin to build foundations for independently drawing these in Year 1. A large emphasis is placed on the part and whole vocabulary.   **Children are not expected to draw part whole models in EYFS. However, they can be introduced by teacher in preparation for Yr 1.**			

Quick Glance: Addition

Year Group	Written Method	Written Method Example
EYFS	Number tracks and Number lines	
Year 1	Number lines Understanding equality	 6 + 5 = ? + 4 6 + ? = 11 6 + 5 = 5 + ?
Year 2	Partitioning column compact column	 Calculations 21 + 42 = 21 + 42
Year 3	Expanded column Formal column	243 +368 611 1 1
Year 4	Formal column	Real life, money, measures. More than 2 numbers added 
Year 5	Formal column	Emphasis on decimals, money, measures, reasoning and worded problems
Year 6	Formal column	Reinforcing and securing all of the above

Year Group	Number size
EYFS	Up to 2 digit + 1 digit
Year 1	Up to 2 digits + 2 digits
Year 2	Up to 2 digits + 2 digits
Year 3	Up to 3 digits (1000)
Year 4	Up to 4 digits including two decimal places
Year 5/6	More than 4 digits and decimals

Year 1 Addition

Objective and Strategies	Concrete	Pictorial	Abstract
Partitioning: Combining two parts to make a whole Number Bonds Expose children, through effective teacher modelling, to continuous models when their number bonds are secure so that their working memory is not overloaded trying to work out the answer and interpret the new structure.	Use cubes to add two numbers together as a group or in a bar.     	 Use pictures to add two numbers together as a group or in a bar.     	3 + 2 = 5 2 + 3 = 5 Fact Families & Number Bonds 4 + 6 = 10 6 + 4 = 10 10= 6 + 4 10 = 4 + 6 8 + 2 = 10 2 + 8 = 10 10= 8 + 2 10 = 2 + 8 5 + 0 = 5 4 + 1 = 5 3 + 2 = 5

Starting at the bigger number and counting on

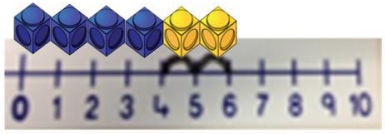
Adding numbers (particularly adding on) could also be shown on number lines with bars above (using Cuisenaire) if the children are confident in their understanding of both number lines and parts/wholes.

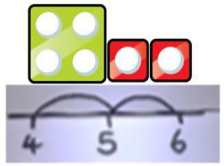
To solve 12 + 5 =

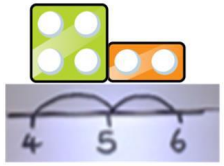


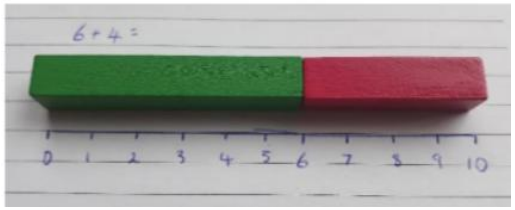
Start with the bigger number. Then count on to the whole amount. What is the total?

Use cubes or Numicon





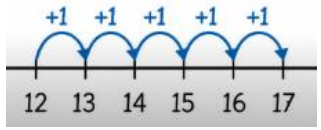




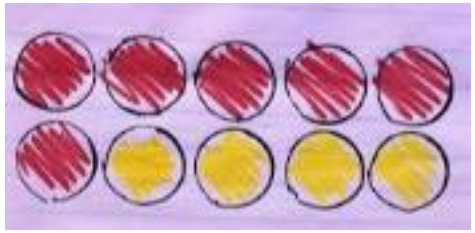
A bar model which encourages children to count on rather than count all. Which are the parts, which is the whole?



Start at the larger number on the number line and count on in ones or in one jump to find the answer.





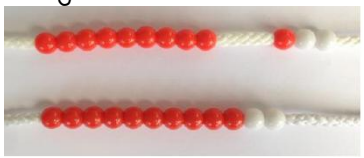
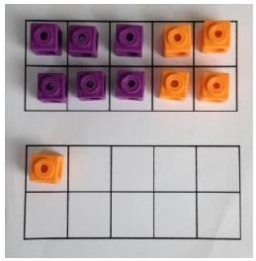
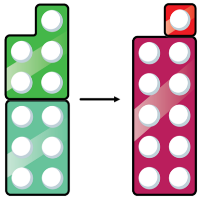
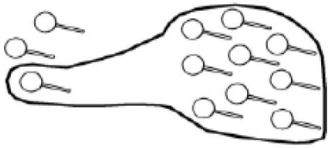
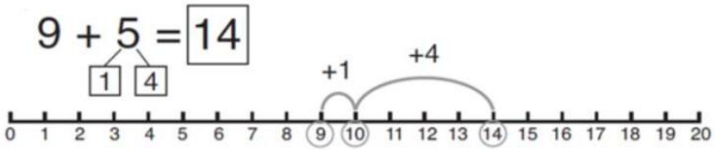
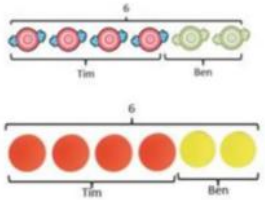
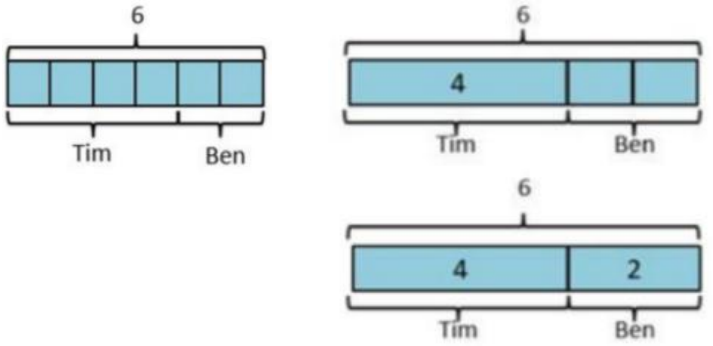




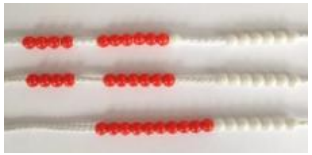
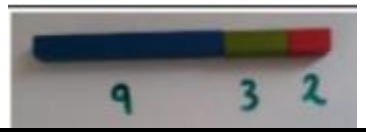
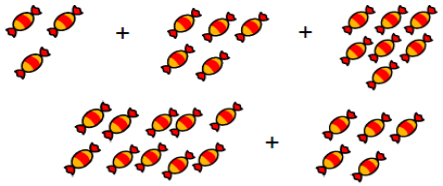
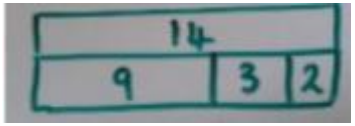
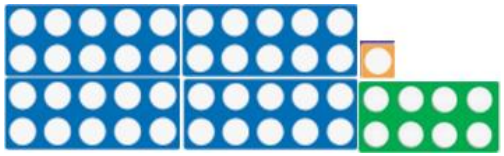
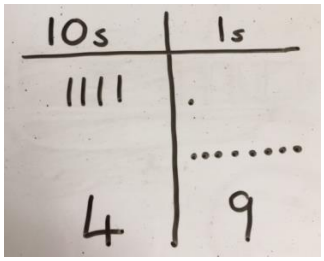
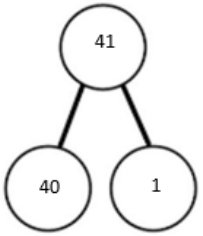
12 + 5 =

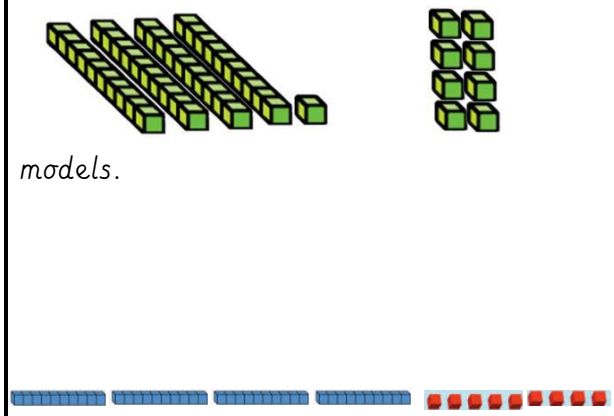
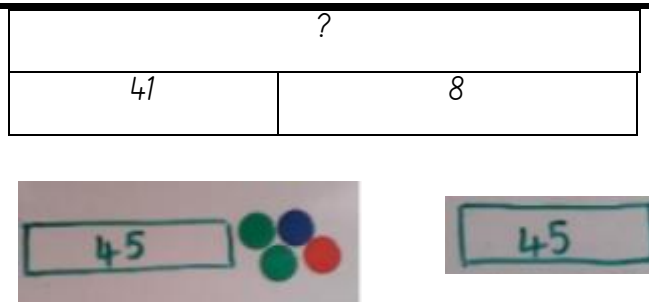
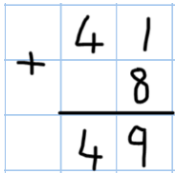
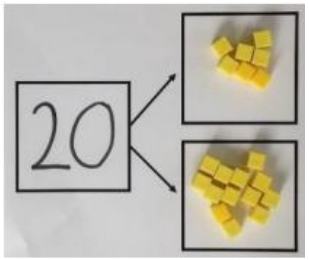
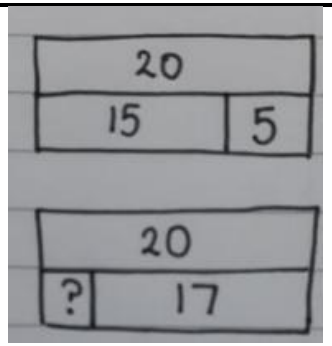
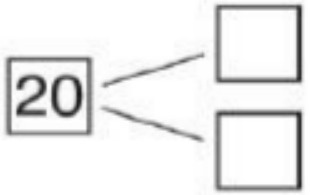
4 + 2 =

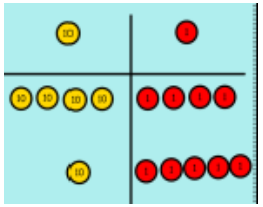
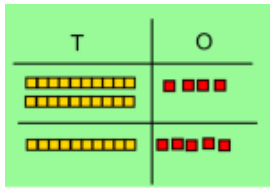
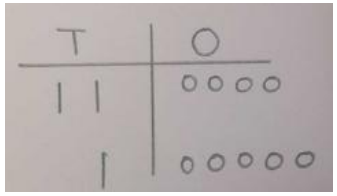
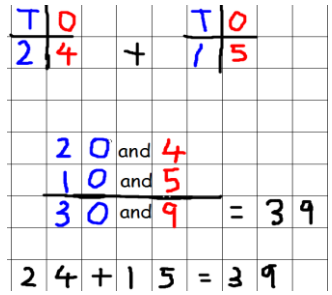
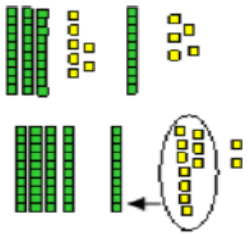
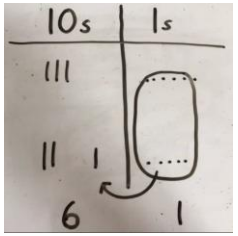
6 + 4 =

<table>
<tr>
<td>Regrouping to make 10.</td>
<td>Using counters/cubes or Numicon

6 + 5 = 11

Start with the bigger number and use the smaller number to make 10.
</td>
<td>Use pictures or a number line. Regroup or partition the smaller number to make 10.

3 + 9 =
</td>
<td>How could we regroup to make a number bond to 10?
3 + 9 =
So...
10 + 2 =</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Small steps

Cubes / counters /Cuisenaire used to transition from real objects to pictorial bars.</td>
<td></td>
<td>There are 6 sweets in a bag for Tim and Ben. If Tim has 4 sweets how many will Ben have?
Use the vocabulary:
4 is a part.
2 is a part.
The whole is 6.</td>
</tr>
</table>

Year 2 Addition

Adding three single digits	Use bead strings to work out sums e.g. 4 + 7 + 6 = 17. Put 4 and 6 together to make 10. Add on 7.  Build a tower of bricks and then ask the child to split them in 3 ways – add the numbers together. 	Add together three groups of objects. Draw a picture to recombine the groups to make 10.  or Bar model represent the 3 numbers. ? / 4 | 6 | 7 	Combine the two numbers to make 10 and then add on the remainder. 4 + 7 + 6 = 10 + 7 10 = 17 Children should be shown missing number sentences.
Tens Ones + Ones	Continue to develop understanding of place value and partitioning e.g. 41 + 8 	Represent base 10 with lines / dots e.g. 41 + 8   The bar model:	41 + 8 Practise adding the ones first. 1 + 8 = 9 40 + 9 = 49

	Using dienes or Cuisenaire rods to show bar models. 		 45 + 4 = 49 4 + 45 = 49
Number Bonds	Children explore ways of making numbers within 20 	 	15 + 5 = 5 + 15 = ? + 17 = 20 17 + ? = 20 20 = 17 + ? 20 = ? + 17 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □

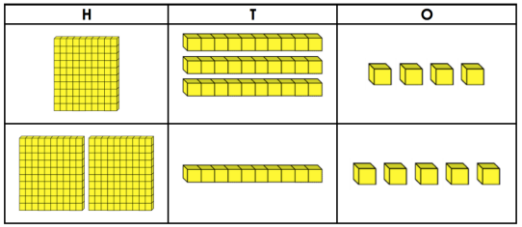
TO+ TO – no regrouping	Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. E.g. 24 + 15   Using dienes/Cuisenaire to show bar models.	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  The bar model: ? | 24 | 15	 Then moving onto… 2 4 +1 5 3 9
TO + TO (With regrouping in the ones)	Show regrouping using dienes: 36 + 25 37 + 15 = 52  Continue to develop understanding of partitioning and place value	Children to represent base 10 by drawing it in a place value chart.  The bar model: ? | 36 | 25	Looking for ways to make 10: 36 + 25= 30 + 20 = 50 6 + 5 = 11 50 + 11 = 61 Formal method: 36 +25 61 1

Year 3 Addition

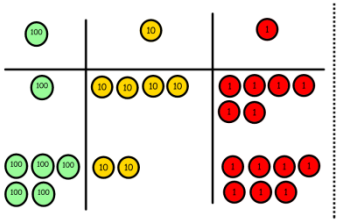
HTO + O (No regrouping)

This can also be done with place value counters or Base 10.

134 + 215 =

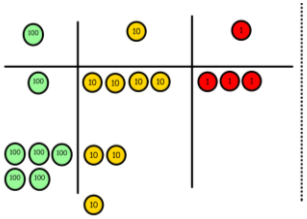


Make both numbers on a place value grid.



146
+ 527

Add up the units and exchange 10 ones for one 10.

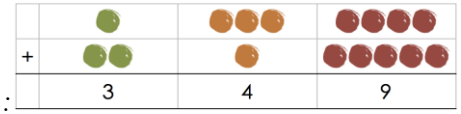


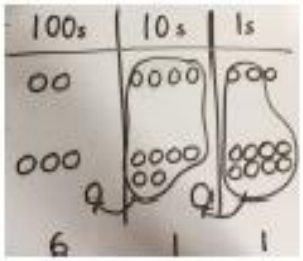
146
+ 527

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value

HTO + O (With regrouping)

Pictorial representation of the columns and counters.





If there are more than 10 or more counters in a column regroup into a new counter in the next column. Then add up all the columns. Make it, Draw it, Write it.

HTO + T (No regrouping)

Bar models

?	
360	25

HTO + TO (With regrouping in the tens)

Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

HTO + HTO (With regrouping in ones & tens)

Introduce the **column method.**

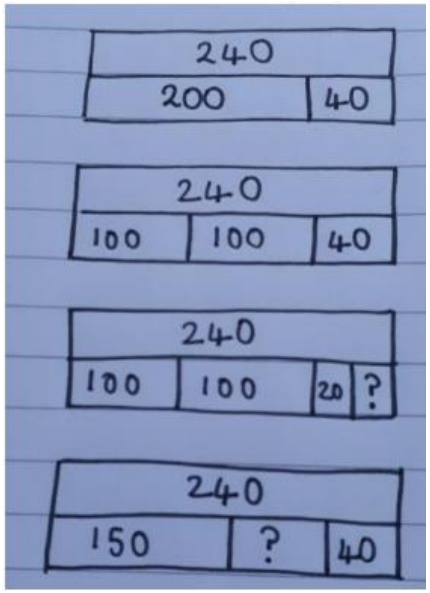
Regrouping:



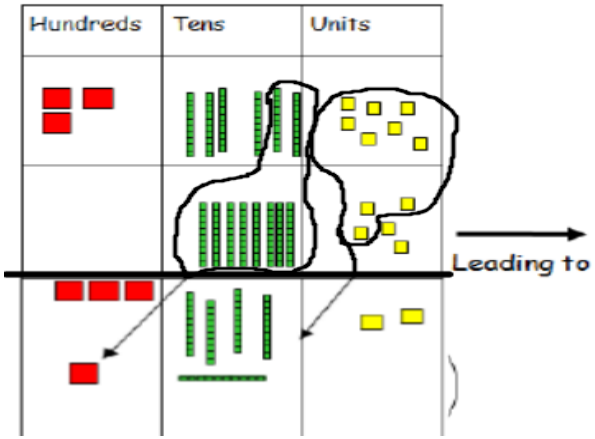
compact column method

up to 3 digits

243
+368
611
1 1

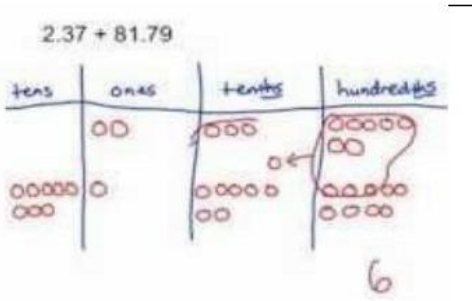
	column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Partition numbers in different ways with the 'unknown' in different places. Use increased levels of proportionality.  	200 + 40 = 240 100 + 100 + 40 =240 100 + 100 + 20 + ? = 240 150 + ? + 40 = 240

Y4 – compact column method up to 4 digits and two decimals (introduced with money) with the same number of digits. If there are more than 10 or more counters in a column regroup into a new counter in the next column. Then add up all the columns.




```
  3 6 7
+ 8 5
-------
  4 5 2
    1 1
```

Y5/Y6 – compact column method with more than four digits and decimals with different place value and regrouping in some columns. Could use place value counters for adding decimals

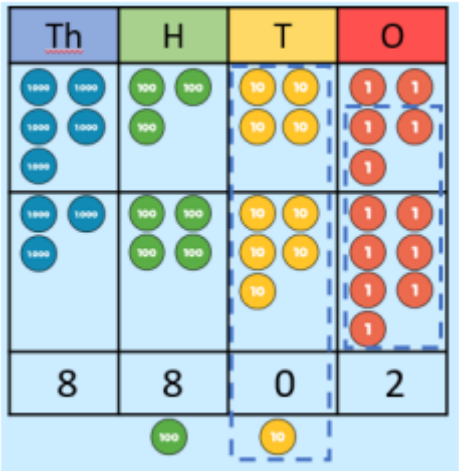


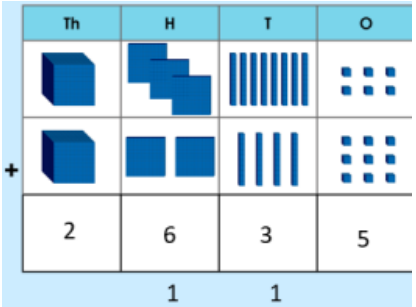

```
  2 3 . 3 6 1
    9 . 0 8 0
  5 9 . 7 7 0
+   1 . 3 0 0
-----------
  9 3 . 5 1 1
  2   1   2
```

```
    8 1,0 5 9
        3,6 6 8
    1 5,3 0 1
+   2 0,5 5 1
-------------
  1 2 0,5 7 9
    1 1 1 1
```

```
    2 3 · 3 6 1
      9 · 0 8 0
  5 9 · 7 7 0
+   1 · 3 0 0
-------------
  9 3 · 5 1 1
  2 1   2
```

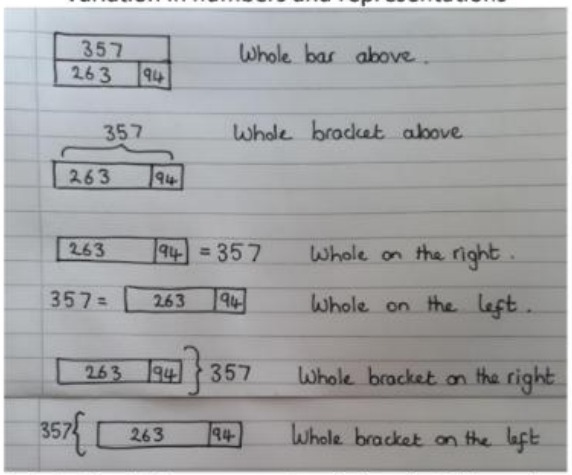
Bar Model examples:





Bar Model examples:

Variation in numbers and representations

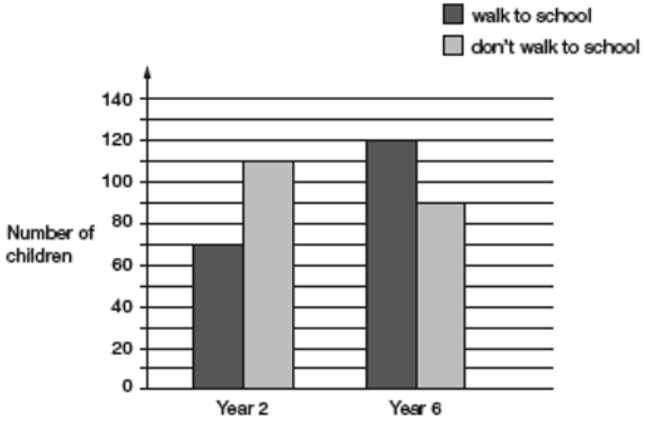


'Whole below' is less conventional though children should understand that it is not incorrect.

Bar Model examples:

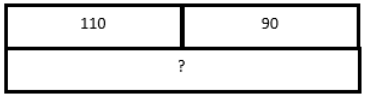
William asks the children in Year 2 and Year 6 if they walk to school.

This graph shows the results.



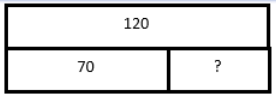
Altogether, how many children don't walk to school?

Altogether, how many children don't walk to school?



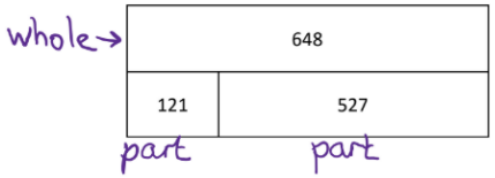
200

How many more Year 6 children than Year 2 children walk to school?



50

Use bar models to understand inverse relationships.



527 + 121 = 648
121 + 527 = 648
648 – 121 = 527
648 – 527 = 121

527 – 121 = 648

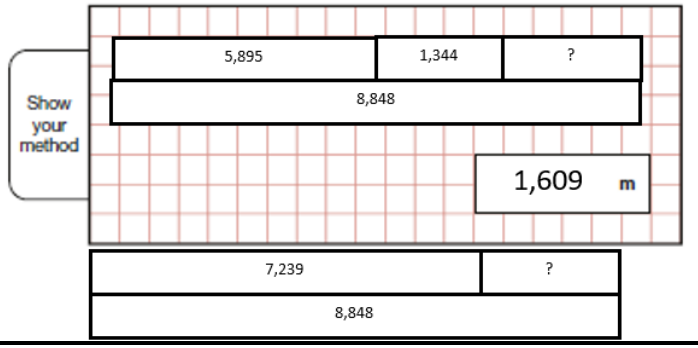
This would NOT be a correct sentence because 527 – 121 would equal 406.

This table shows the heights of three mountains.

Mountain	Height in metres
Mount Everest	8,848
Mount Kilimanjaro	5,895
Ben Nevis	1,344

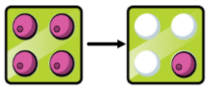
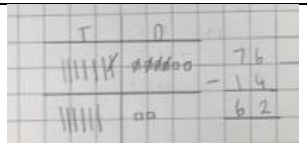
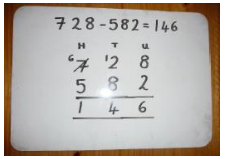
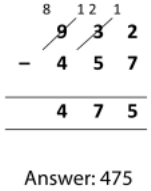
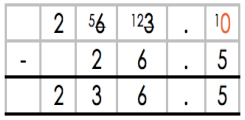
How much higher is Mount Everest than the combined height of the other two mountains?

Show your method



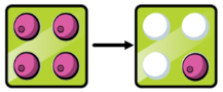
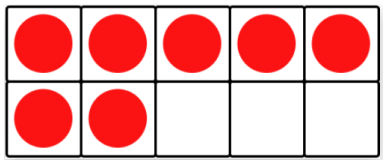
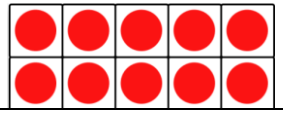
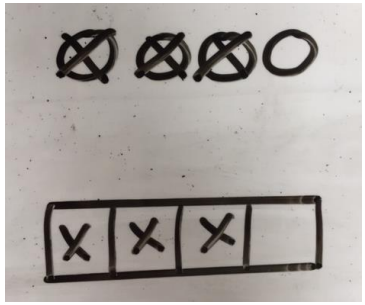
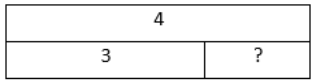
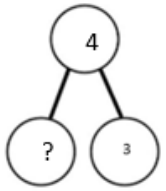
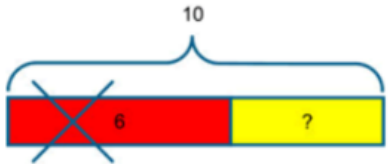
2 marks

Quick Glance: Subtraction

Year Group	Written Method Name	Written Method Example
EYFS	Taking away ones and Number lines	 4 - 3 = 1
Year 1	Number lines and Informal Partitioning	Children start to show recognisable abstract number sentences. 13 - 7 = 6, 13 – 6 = 7
Year 2	Partitioning column	
Year 3	Exchanging Formal column	
Year 4	Noughts Formal column	Real life, money, measures, 700 - 487 
Year 5	Formal column	Emphasis on decimals, money, measures, reasoning and worded problems 
Year 6	Formal column	Reinforcing and securing all of the above

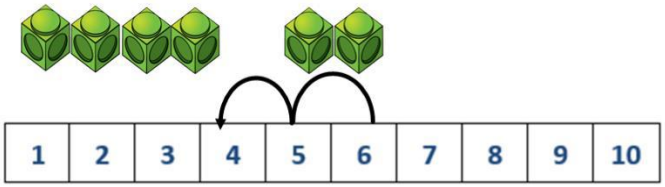
Quick Glance Subtraction Number Size	
Year Group	Number size
EYFS	Up to 1 digit - 1 digit
Year 1	Up to 2 digits - 1 digit
Year 2	Up to 2 digits - 2 digits
Year 3	Up to 3 digits (1000)
Year 4	Up to 4 digits including two decimal places
Year 5/6	More than 4 digits and decimals

Year 1 Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes, Numicon and other items such as bean bags etc to show how objects can be taken away.  4 - 3 = 1  7 – 4 =  Use counters and tens frame to work though problems step by step. E.g. We had 10 pencils. How many did we give away (6)? How many left? Reinforce 6 is a part and 4 is a part. 	Cross out drawn objects to show what has been taken away. Bar model can be used.    Discrete model 7 -4  	Children start to show recognisable abstract number sentences. 4 – 3 = 1 □ = 4 - 3 7 – 4 = 3 **I had 10 pencils and I gave 6 away, how many do I have now?** (This time we know the whole but only one of the parts, so the whole is partitioned and one of the parts removed to identify the missing part) 10 – 6 = 6 + ? = 10

Counting back

Children to first use cubes on top of number lines or number tracks before

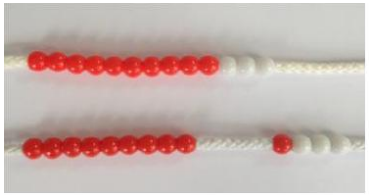


jumping on a number line

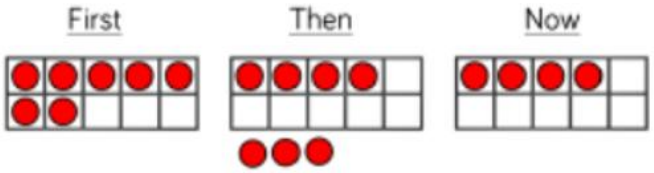
6 – 2 = Start with 6 and count back



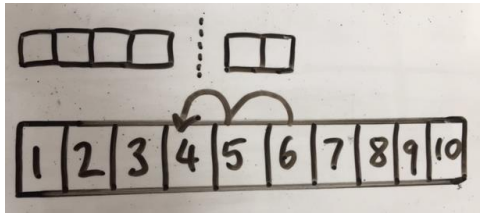
Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.



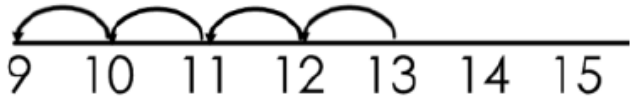
Count back using tens frame and counters.



Children represent what they see pictorially.



Start at the bigger number and count back the smaller number showing the jumps on the number line.





This can progress all the way to counting back using two 2-digit numbers.

Children start to show recognisable abstract number sentences.

6 – 2 = 4

Put 13 in your head, count back 4. Use your fingers to help.

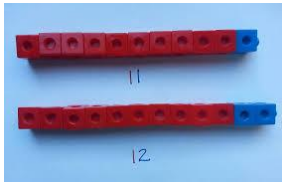
Children to then move on to mentally subtracting 3 (to 10) and then subtracting 1.

13 – 4 = 9

7 – 3 = 4

Find the difference

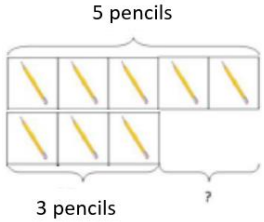
Compare amounts and objects to find the difference.



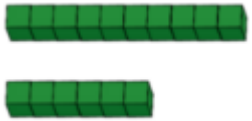
Use cubes to build towers or make bars to find the difference

Use basic bar models with items to find the difference

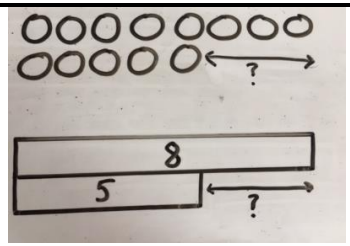
Use a tens frame with physical objects.

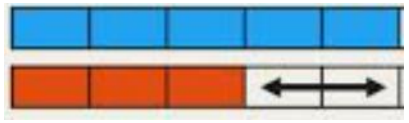


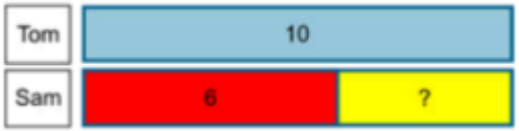
Use cubes/counters/Dienes to build the numbers.



Draw cubes/other concrete objects or use the bar model to illustrate what they need to calculate







Find the difference between 8 and 5

8 – 5 =

Explore why 9 – 6 = 8 – 5

Hannah has 8 sandwiches, Helen has 5 sandwiches. Find the difference between the number of sandwiches.

Sam has 5 pencils and Tim has 3 pencils. How many more pencils does Tim have?

Tom has 10 pencils and Sam has 6 pencils. How many more does Tom have?

(The bar is particularly valuable for seeing the difference between the two quantities)

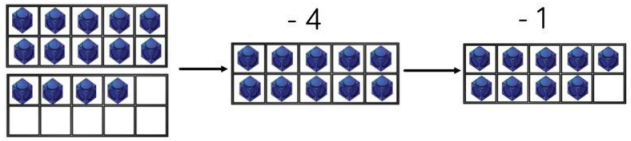
What's the difference between 10 and 6?

The difference between 10 and 6 is ___

10 – 6 = ___

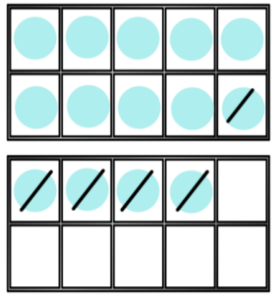
Make 10

14 - 5 (Numicon, counters, 10 square, bead string)



Make 14 On the tens frame
Take away the 4 first to leave 10
Then takeaway 1 so you have taken away 5.
You are left with the answer of 9.

Children present the tens frame pictorially and discuss what they did to make 10.



Show how to make 10 by partitioning

14 - 5 = 9

4 1

14 – 4 = 10
10 – 1 = 9

Children start to show recognisable abstract number sentences.

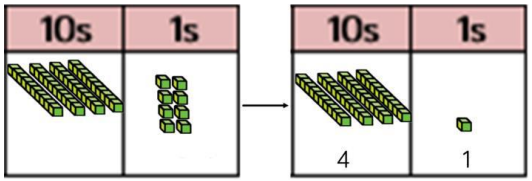
13 - 7 = 6, 13 – 6 = 7

Year 2 Subtraction

TO – O (No exchanging)

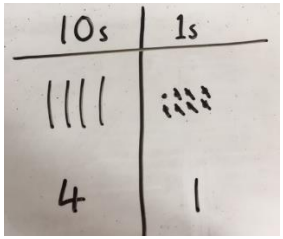
Create the bigger number using dienes/place value counters and then subtract the smaller number.

48 – 7

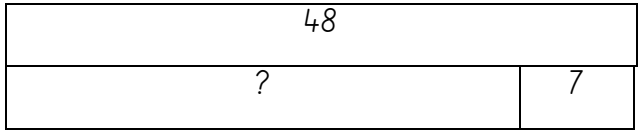


Children should be advised to use mental methods to calculate this sum initially, before proving their answer with written methods.

Draw the dienes/place value counters and then cross out what you are subtracting.

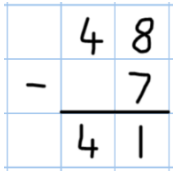


The bar model:

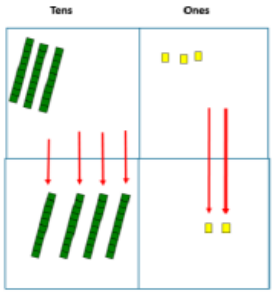


Count back 7 or use column method

48 – 7 =

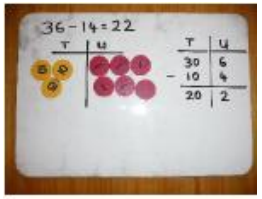


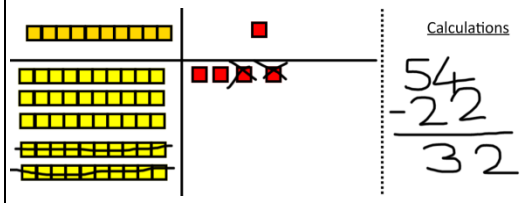
TO - TO Column method (without exchanging)

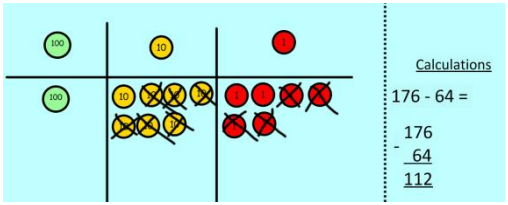


Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.





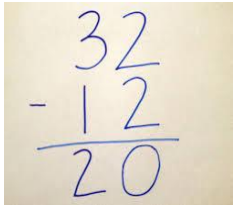


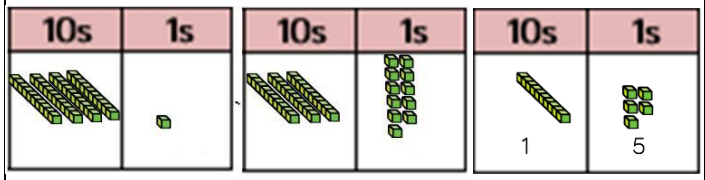
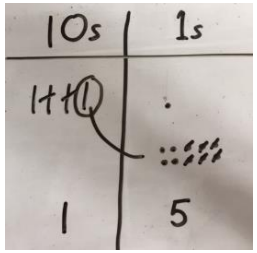
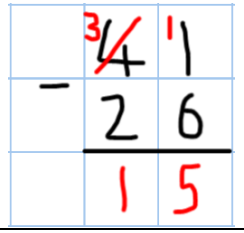
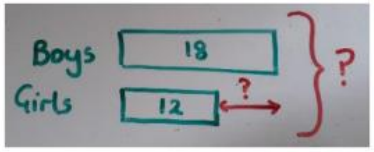
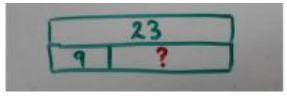
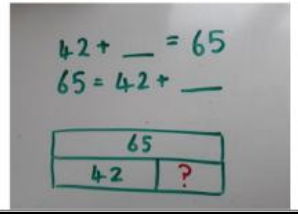
Draw the Base 10 or place value counters alongside the written calculation to help to show working.

47 – 24 = 23

40 + 7
– 20 + 4
20 + 3

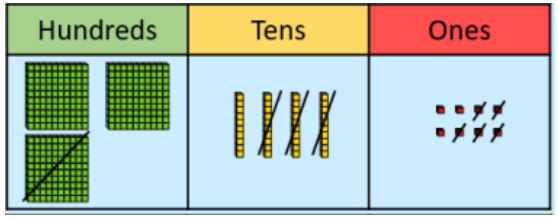
This will lead to a clear written column subtraction.



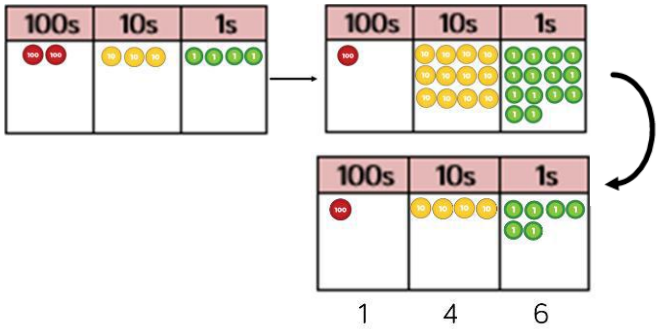
TO - TO Column method with exchanging	Create the bigger number using dienes 41 - 26 	Draw the dienes and then cross out what you are subtracting. The exchanging must be clearly shown. 41 - 26 	Introduction of the **column method:** Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 
Find the difference		Use comparison continuous models to find the difference and also to find the whole.  	18 – 12 = 6
Missing number problems	Once children are using the bar model with the whole as a bar at the top in Year 2, they can begin using bar models to represent missing number problems providing they have a secure understanding of how to interpret the parts, the whole and the unknown part of the question.	 	___ - 9 = 23 23 = ___ - 9 42 + ___ = 65 65 = 42 + ____

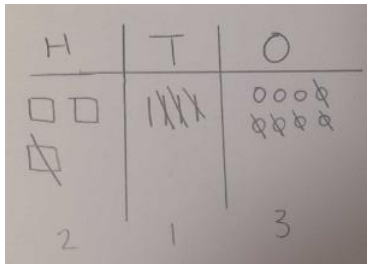
Year 3 Subtraction

HTO - O (No exchanging)	Use dienes to start with before moving on to place value counters.	Draw representations to support understanding, Dienes or place value counters.	**Formal column method** with exchanging. Children must understand what has happened when they have crossed out the digits by using correct vocab This will lead to an understanding of subtracting any number including decimals.
HTO - O (With exchanging)			
HTO - TO (No exchanging)			
HTO - TO (With exchanging in the tens)	Start with one exchange before moving onto subtractions with 2 exchanges.		
HTO - TO (With exchanging in ones & tens)			



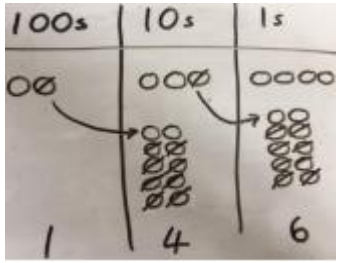
234-88





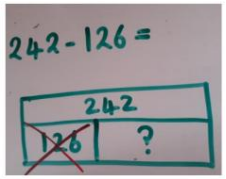
Represent the place value counters pictorially; remembering to show what has been exchanged.

234-88

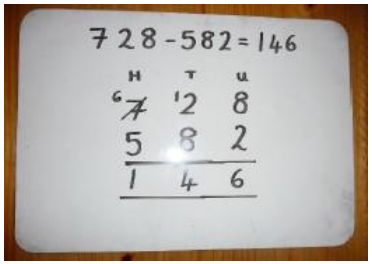


The bar model:

234	
?	88



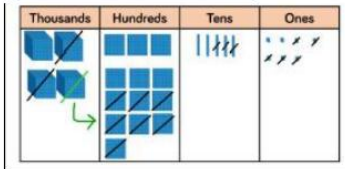
	H	T	O
	3	4	8
−	1	3	5
	2	1	3



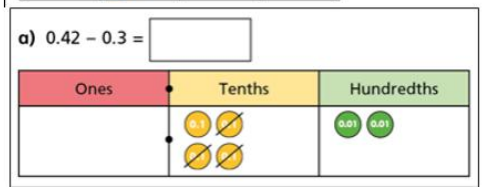
Y4 – compact column method with regrouping with more than 3 digits. Include 0s. Make it, Draw it, Write it. Subtraction with money (decimals).

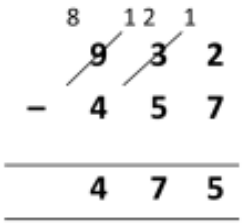
Model process of exchange using base ten or PV counters.

4,357 – 2,735 = 1,622

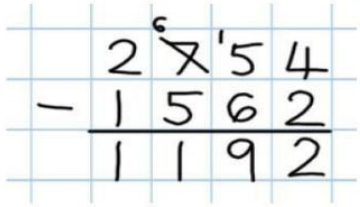


a) 0.42 – 0.3 =





Answer: 475



3 1
4357
– 2735
1622

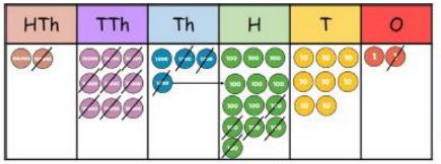
Ensure the children write out the column method alongside the concrete resources.

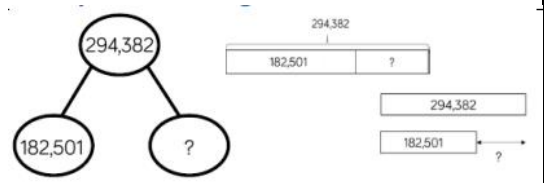
Y5 + Y6 compact column method with regrouping with more than 4 digits.

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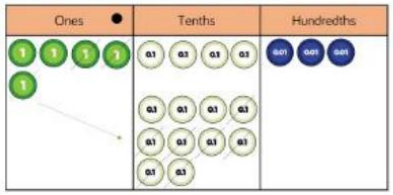
Place value counters for decimals with different amount of decimal places.

294,382 – 182,501 = 111,881

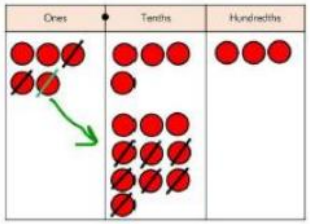




5.43 – 2.7 = 2.73



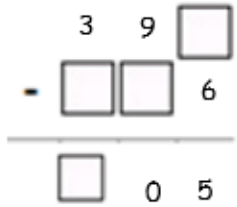
Children to draw pv counters and show their exchange—see Y3



263 – 26.5 =

Missing digits:

	2	5 6	12 3	.	10
-		2	6	.	5
	2	3	6	.	5

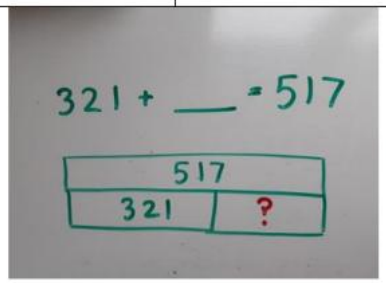


Bar Model: Addition and subtraction – missing number problems

Once children are using the bar model with the whole as a bar at the top in Year 2, they can begin using bar models to

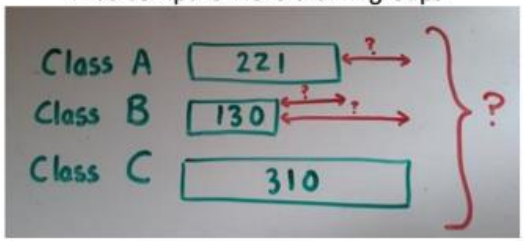
represent missing number problems providing they have a secure understanding of how to interpret the parts, the whole and the unknown part of the question.

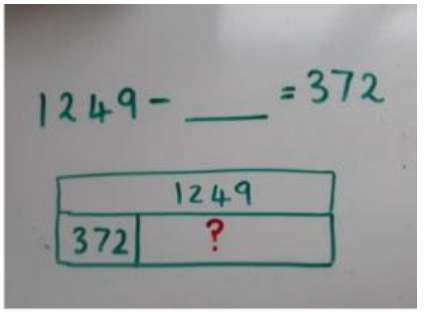
Year 3 | Year 4



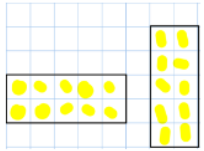
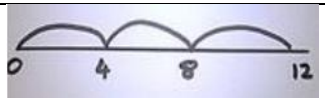
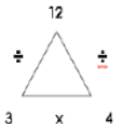
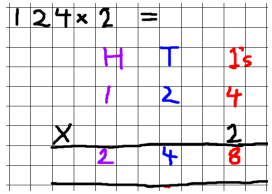
Bar Model: Addition and subtraction – missing number problems

Use comparison continuous models to find the difference, find the whole with numbers ≥ 3 digits.
Also compare more than 2 groups.

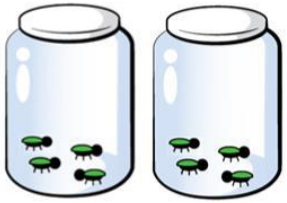
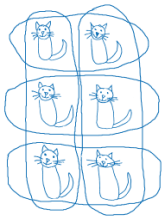
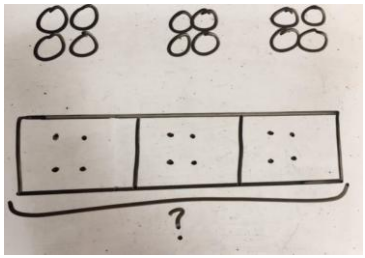




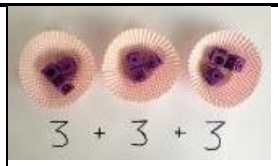
Quick Glance: Multiplication

Year Group	Written Method Name	Written Method Example
EYFS	Arrays	
Year 1 Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples,	Arrays and repeated addition	
Year 2 Recall multiplication facts, and corresponding division facts, in the 10, 5, 2	Arrays in a grid Introducing inverse partitioning	3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3 
Year 3 Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8	Formal short multiplication	23 × 3 69
Year 4	Expanded short multiplication Formal short multiplication Multiplying by 10, 100, 1000	Compact Short: 
Year 5	Formal long multiplication including whole numbers by decimals Multiplying decimals by 10, 100, 1000	Compact Long: 3 2 X 2 4 1 2 8 (4 x 32) + 6 4 0 (20 x 32) 7 6 8 (24 x 32) 124 × 26 becomes 1 2 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1
Year 6	Formal long multiplication Including onto multiplying decimals by decimals	1.2 4 X 2.6 7 4 4 + 2 4 8 0 3.2 2 4

Year 1 Multiplication

Objectives and strategies	Concrete	Pictorial	Abstract
Doubling / repeated grouping	Use a set of objects. Double the set by finding the same number again.  4 + 4 double 4 is 8 4 x 2 = 8  3 x 4 4 + 4 + 4 There are 3 equal groups with 4 in each group.	Draw the objects and use bar models  showing: 2 x 3 and 3 x 2  3 x 4 and 4 x 3	Children may start to show recognisable abstract number sentences. 3 x 4 = 12 4 + 4 + 4 = 12

Counting in multiples/ repeated addition

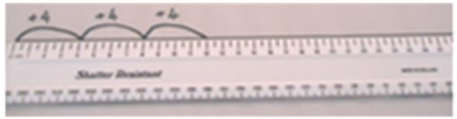


There are 3 equal groups with 3 in each group.

3 x 4 (3 lots of 4)

4 x 3 (4, 3 times)

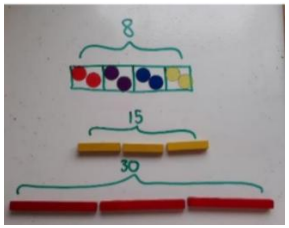




Use a bead string to show repeated addition.



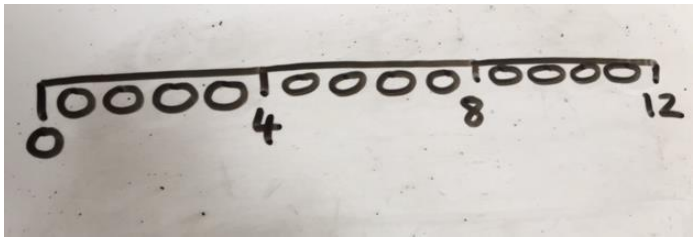
Children use counters and Cuisenaire Rods to partition totals into equal parts.

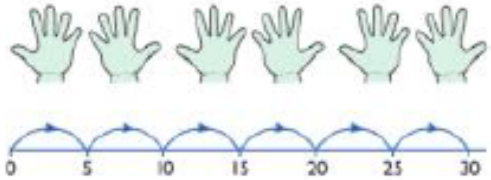


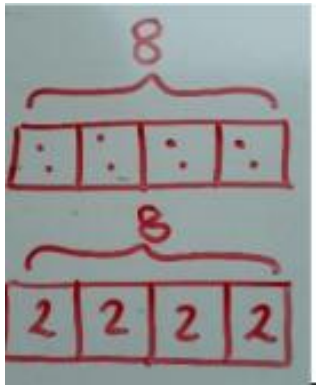
Using Numicon to show 3 x 5:



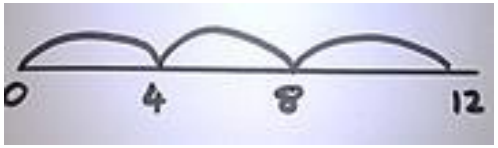
Represent on the number line. Represent pictorially alongside a number line.







Abstract number line showing three jumps of four.



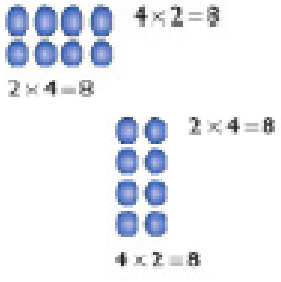
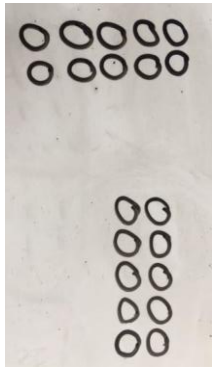
Children count in multiples of a number out loud.

Write sequences with multiples of numbers. 2, 4, 6, 8 etc

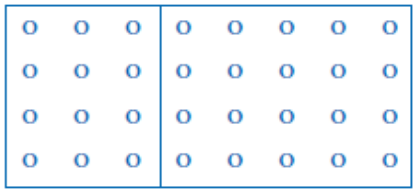
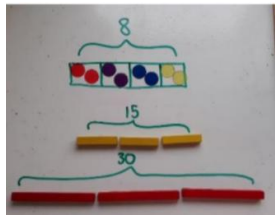
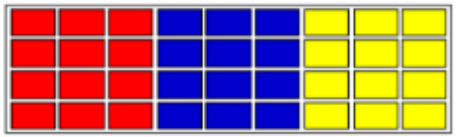
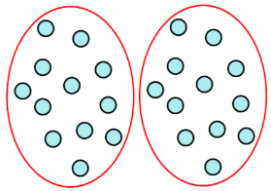
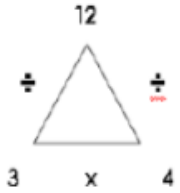


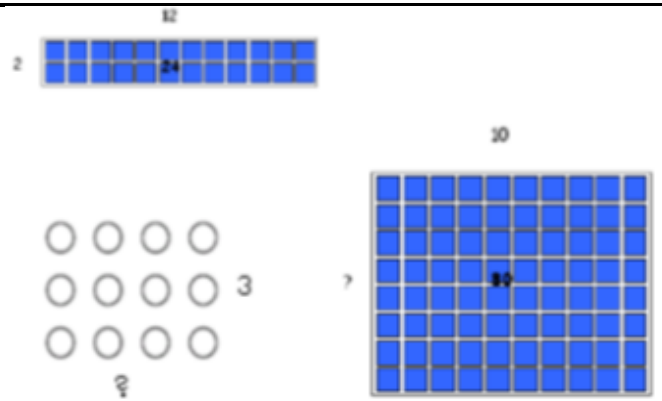
Children are taught about the multiplication 'x' symbol.

3 x 4 = 12 is the same as 4 + 4 + 4 = 12

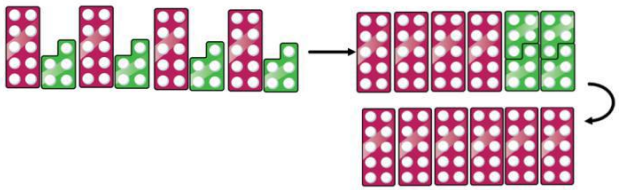
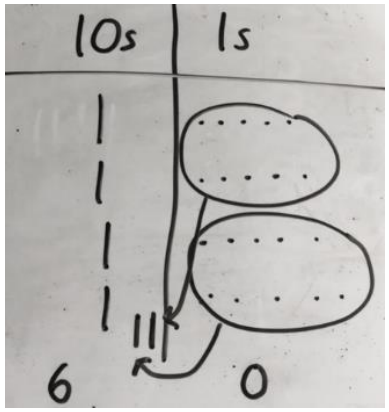
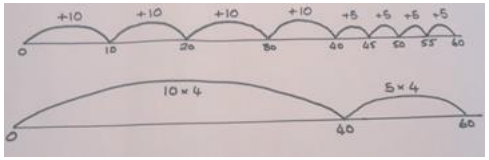
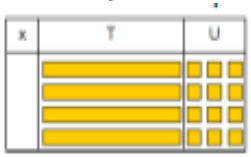
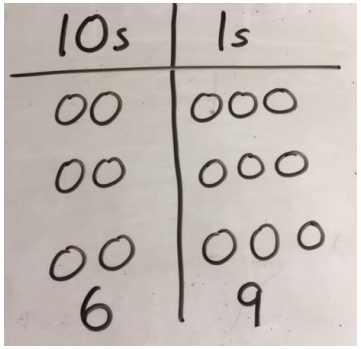
Introduction of using arrays to count in multiples of 2, 5, 10 (commutative law)	Use a set of objects. Children can place them in groups or start to focus them in on array shapes. 2 × 6 6 × 2 	Draw the objects in arrays. Draw in different rotations to find the commutative sentences. This prepares children for finding factors. Also, to help find the area of rectangles.  	Children start to use an array to write a range of abstract calculations. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 =10 10 = 5 + 5

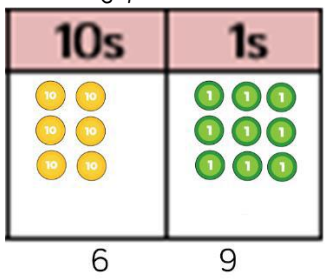
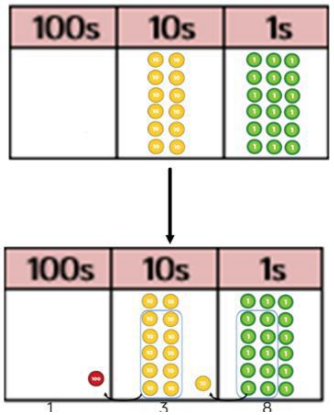
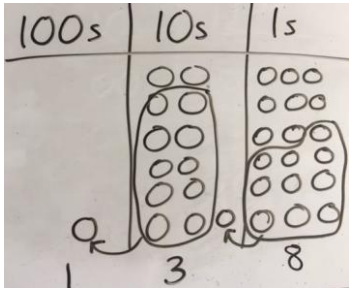
Year 2 Multiplication

Consolidating use of arrays and repeated addition (distributive law)	32 pegs on a board are to be arranged into fours. How can these be shown? This shows the distributive law where 8 x 4 = 3 x 4 + 5 x 4.  Follow Year 1 sequencing using Cuisenaire rods/counters/cubes and progressing to use continuous models using bars for the top whole. 	Ch to illustrate this in different ways and should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks.  9 x 4 = 3 3 3 4 Draw the parts first as you count up in the number: 5 5 5 5 5 Then add the whole bar on top: 25 5 5 5 5 5	Which could also be seen as 9 x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 + 12 = 36 Or 3 x (3 x 4) = 36 5 x 5 = 25 = 5 x ___
Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 2 x ? = 24 Sharing: 	Drawing arrays or groups: 3 X ? = 12	Introducing the Inverse operations Trios can be used to model the 4 related multiplication and division facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3 

			Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

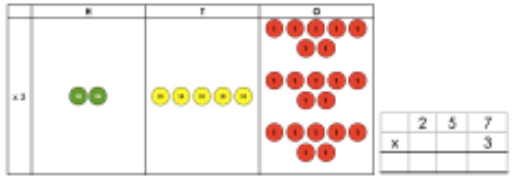
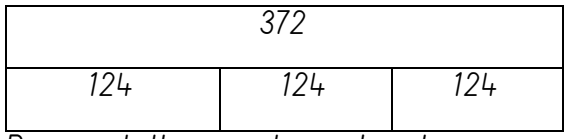
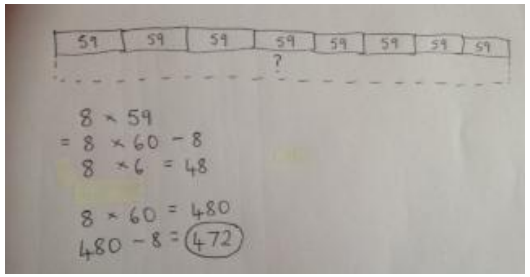
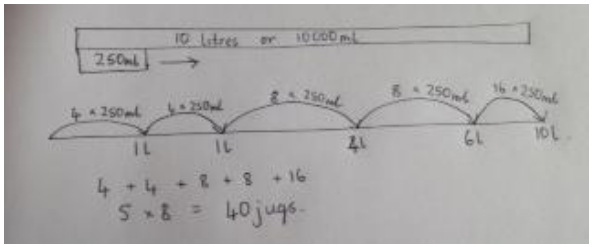
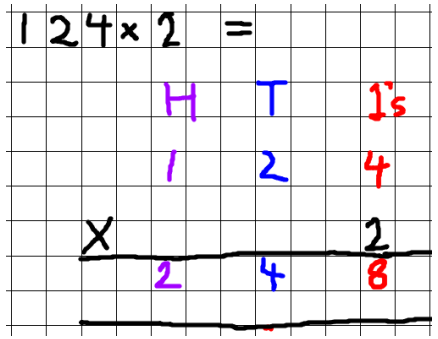
Year 3 Multiplication

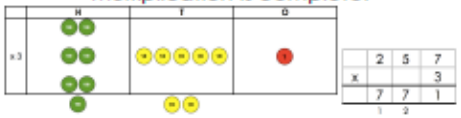
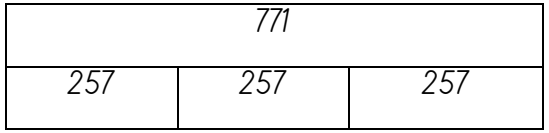
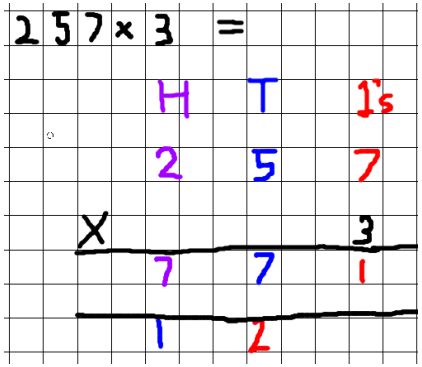
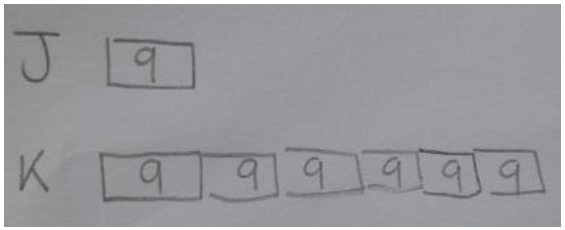
Partition to multiply	Use Numicon to show 15 x 4  Show multiplication of the 10s and the 1s separately	Children represent the concrete manipulative in a picture 	Children be encouraged to show the steps they have taken: 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used: 
Formal column method TO x O (No regrouping)	1) Show the link with arrays with unifix 13 x 4 2) Using Dienes in a grid  4 rows of 13	Children can represent the work they have done in a way that they understand.  Show in a bar model.	**Introduction** with expanded short multiplication: Children to record what it is they are doing to show understanding.

	3) Using place value counters  	52 4 4 4 4 4 4 4 4 4 4 4 4 4 52 = 4 x ___	23 23 × 3 x3 69 9 (3x3) + 60 (20x3) 69 (23x3)
TO x O (with regrouping of ones into tens)	Formal column method with place value counters. 6 x 23  	Children to represent the counters/base 10, pictorially e.g. the image below.  	Discuss how multiplying 6 x 2 gives you 12 and the answer is ten times bigger. 23 X 6 138 1 1

Problem solving using bar models (using multiples of 2,3,4, 5,8 and 10)	Peter has 4 books Harry has five times as many books as Peter. How many books has Harry?  	This could be first introduced using counters/cubes/Cuisenaire rods.  	Further questioning: How many more does Harry have than Peter? How many fewer does Peter have than Harry? How many do they have altogether? 4 x5 =

Year 4 Multiplication

HTO x O (no regrouping) HTO X O (extra digit in the answer) HTO X O (with regrouping of ones into tens) HTO X O (with regrouping of tens into hundreds) HTO X O (with regrouping of ones into tens and tens into hundreds)	Children can continue to be supported by place value counters at the stage of multiplication. 257 x 3 = Use the place value counters to demonstrate multiplying in columns. Make the number with the place value counters.  Start with the ones. Make 3 groups of 7. E.g. 3 x 7 = 21  If there are 10 or more counters in a column exchange for counters in the next highest column. E.g. 20 ones for 2 tens 	Children to represent the counters / dienes pictorially: With regrouping of 1s into Tens, using dienes: Show in a bar model.  Bar modelling and number lines can support learners when solving problems with  multiplication alongside the formal written methods. 	Introduction of **formal short multiplication:** 

	Now look at the tens, children make 3 groups of 50. E.g. 3 x 50 = 250  If there are 10 or more counters in a column exchange for the next highest column. E.g. 10 tens for 1 hundred The previously exchanged counters are added to the product.  Now look at the hundreds, make 3 groups of 200. E.g. 200 x 3 = 600 The previously exchanged counters are added to the product and the multiplication is complete.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens, then hundreds etc.	Show in a bar model. 	With regrouping 
Problem solving using bar models	Jack has 9 books Kim has 6 times as many books as Jack. How many books does Kim have? 	This could be introduced using counters/cubes/Cuisenaire rods. 	Further questioning: How many more does Kim have than Jack? How many fewer does Jack have than Kim? How many do they have altogether? 9 x6 =

Y5 - Introduction of **long multiplication** with up to 4 x 2 digits. reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

```
  3 2
X 2 4
-----
  1 2 8  ( 4 x 32)
+ 6 4 0  (20 x 32)
-----
  7 6 8  (24 x 32)
```

Onto more compact methods

Y5 - Introduction of **compact long multiplication** with up to 4 x 2 digits.

Onto multiplication of decimals.

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

124 x 26 becomes

```
  1 2 4
    X 2 6
-------
    7 4 4
+ 2 4 8 0
-------
  3 2 2 4
  1   1
```

```
  1 2 4
    X 2.6
-------
    7 4.4 ( 0.6 x 124)
+ 2 4 8.0 ( 2   x 124)
-------
  3 2 2.4 (2.6 x 124)
```

When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract

Y6 – Consolidation of **compact short multiplication** and **compact long multiplication** methods with up to 4 digits by a 2 digit.

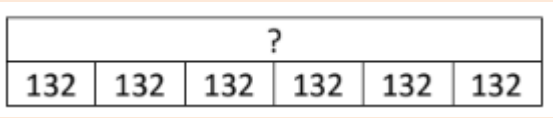
Also onto **multiplication of decimals by decimals** – estimation first, then adding no of decimal places.

```
  1.2 4
  X 2.6
-------
    7 4 4
+ 2 4 8 0
-------
  3.2 2 4
  1   1
```

UKS2 Bar Models

Use the structure of repeated addition bar models to help understand and represent questions but use formal written methods to calculate answers. For calculations such as 43 x 28, a bar model would not be suitable. This is an arithmetic question and best suited for short multiplication. Bar models could be used to represent problems such as:

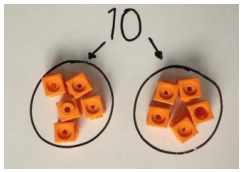
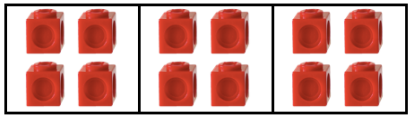
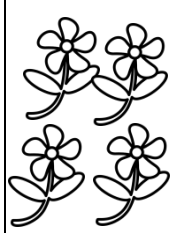
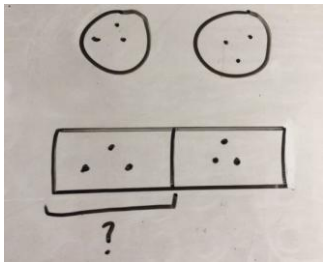
Irvin bought 6 bags of apples, each weighing 132kg.

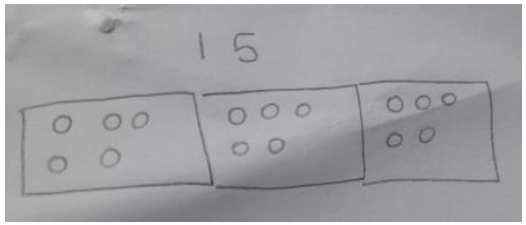


Quick Glance: Division

Year Group	Written Method Name	Written Method Example
EYFS Year 1	Sharing and grouping in circles	Sharing: 3 3 3 3 Grouping:
Year 2	Sharing and grouping in arrays	96 ÷ 3 = 32
Year 3	Sharing within place value columns Remainders	42 ÷ 3 = 14 17 ÷ 3 = 5 r 2
Year 4	Compact short division	Compact short division: : 5 | 7 22 28 — 1 4 5 r3
Year 5	Compact short division Remainders - or written as mixed number / decimal	145 r3 or 145 2/5 or 145.6
Year 6	Long division	Long division:

Year 1 Division

Objectives and strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups Share objects into groups. I have 12 cubes.  Can they be shared equally in 3 groups? After sharing between 3 groups we have found that are 4 in each group.  12 ÷ 3 = 4	Children use pictures or shapes to share quantities.    8 ÷ 2 = 4  12 ÷ 3 = 4	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts. Share 9 buns between three people. 9 ÷ 3 = 3 Share 12 sweets between 3 people. 12 ÷ 3 = 4

Division as grouping	Divide quantities into equal groups.  	. 	Divide 15 into groups of 3 . How many are in each group? After making groups of 5 we discovered there were 3 of them.

Year 2 Division

Sharing objects into equal groups

Division symbol

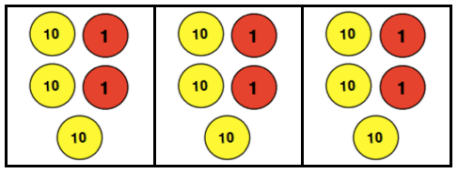
e.g.

20 ÷ 5 =

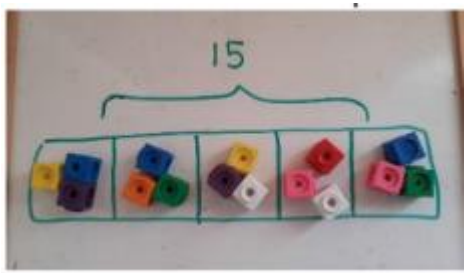
Children can choose their preferred method but if unsure, the sharing method should be favoured until their counting in multiples is secure enough to support grouping.

If the number becomes large, choosing the most efficient method is important.

Using place value counters e.g. 96 ÷ 3 = 32

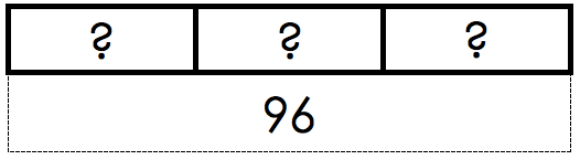


After sharing we found there were 3 tens and 2 ones in each group.



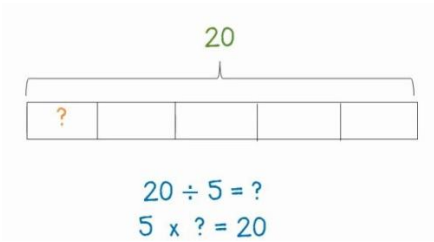
Use pictures or shapes to share quantities.

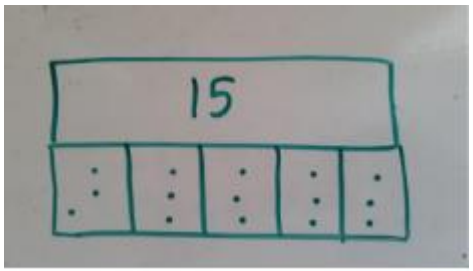
Bar Modelling:
Split the bar into the number of groups you are dividing by and work out how many would be within each group.



96 ÷ 3 =

Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group

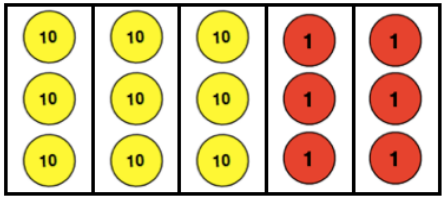




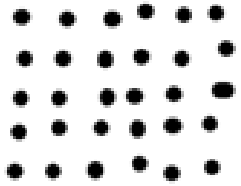
Share £96 between 3 children.
£96 ÷ 3 = £32

15 cubes shared between 5 friends.
Show 15 as the whole bar.
Split the bottom bar into 5, 1 part for each friend.
Count out the 15 across each part – remember division must be equal parts.

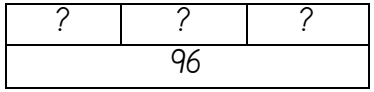
<table>
<tr>
<td>Grouping objects</td>
<td>Divide quantities into equal sized groups. Use cubes, counters, objects or place value counters to aid understanding.
I have 12 cubes.

Using place value counters e.g. 96 ÷ 3 = 32


After making groups of 3, we find there were 3 groups of ten and 2 groups of one.</td>
<td>Represent using arrays: How many strawberries will each child have if 30 are shared between 5 children?

30 ÷ 6 = 5

30 ÷ 5 = 6

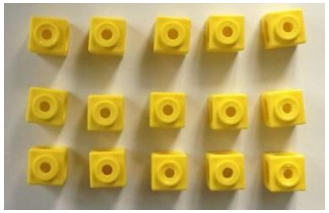
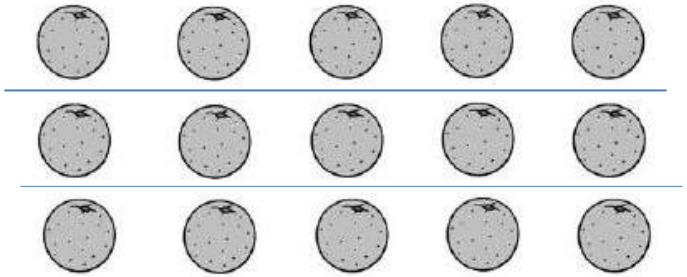
Bar Modelling:
You know how many would be within each group, but need to find out how many groups.



96 ÷ ? = 3</td>
<td>Sweets are sold in bags of 3. If I have 12 sweets how many bags would I need?

12 ÷ 3 = 4

There are 96 children sitting in rows of 3.
How many rows are there?

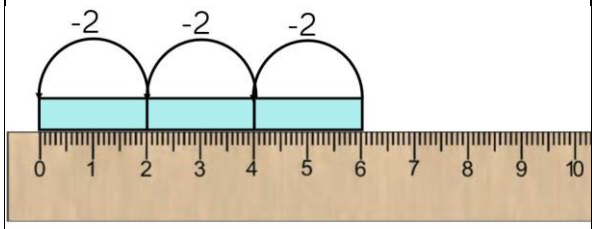
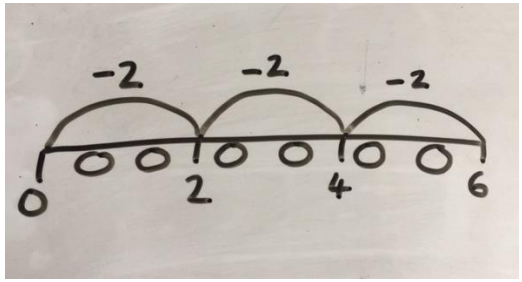
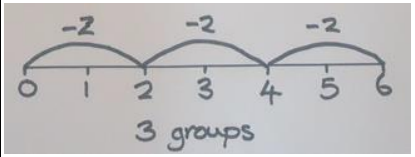
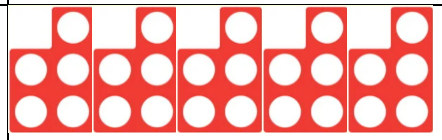
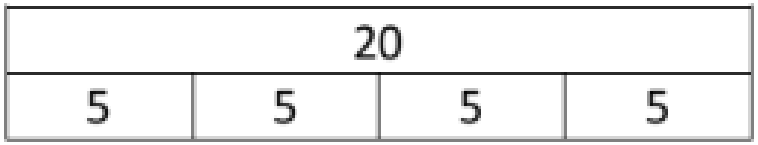
96 ÷ 3 = 32</td>
</tr>
<tr>
<td>Division within arrays</td>
<td>
Link division to multiplication by creating an array and thinking about the number sentences that can be created.</td>
<td>
Draw an array and use lines to split the array into groups to make multiplication and division sentences.</td>
<td>Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28

4 x 7 = 28

28 ÷ 7 = 4

28 ÷ 4 = 7</td>
</tr>
</table>

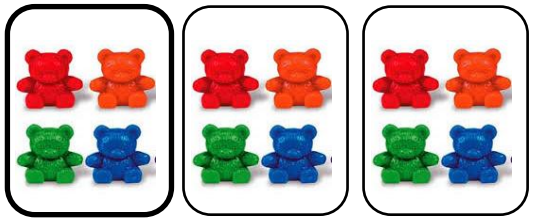
	Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15		
Repeated subtraction	Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted 
Solving missing number problems: part unknown			20 ÷ ___ = 5 Children will be taught to solve this by counting up in 5s, thinking about how many groups of 5 it is.

Year 3 Division

Division with a remainder

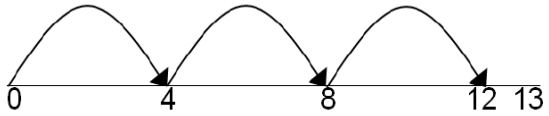
14 ÷ 3 = 4 r 2

Divide objects between groups and see how much is left over





Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



13 ÷ 4 = 3 r 1

Draw dots and group them to divide an amount and clearly show a remainder.



Bar model can be used to display whole being divided into equal parts with a remainder left over.

5	5	5	R2
17			

Complete written divisions and show the remainder using r.



14 ÷ 4 = 3 r2

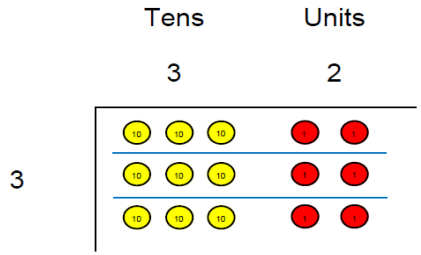
17 ÷ 3 = 5 r 2

Short division

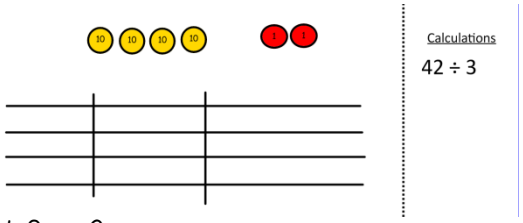
No remainder, Exchanging

Remainder, no carrying

HTO ÷ O
(No regrouping and no remainder)



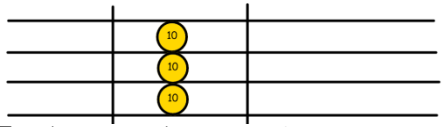
Use place value counters to divide using the bus stop method alongside



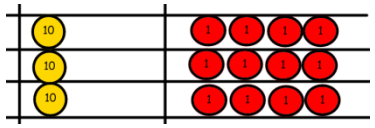
42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



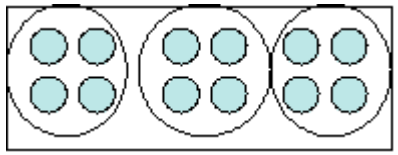


Exchange this ten for ten ones and then share the ones equally among the groups.

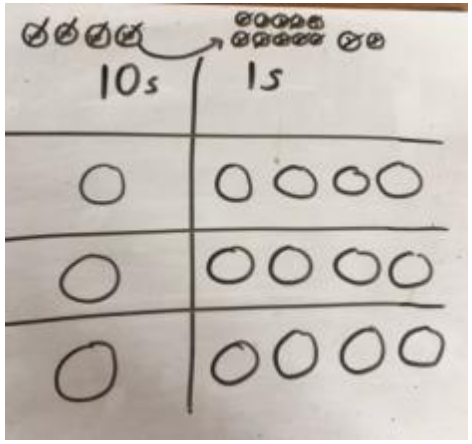


We look how much in 1 group so the answer is 14

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

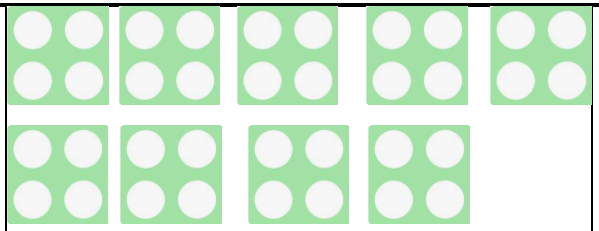
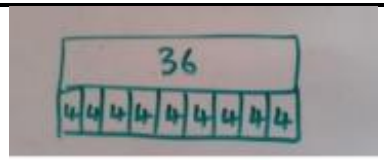
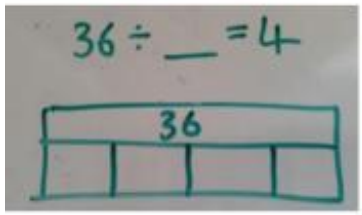
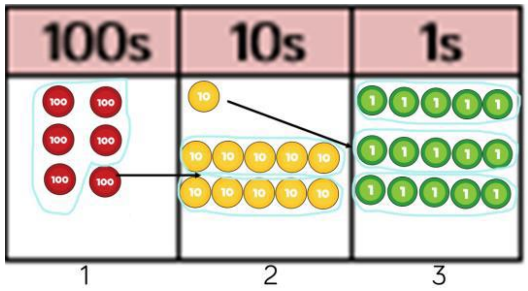
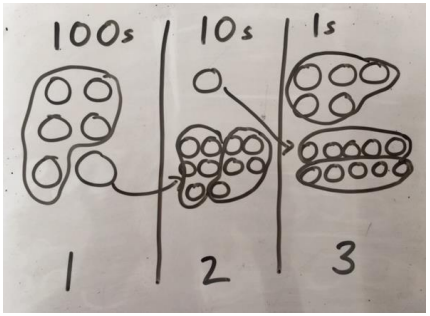


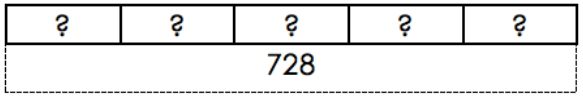
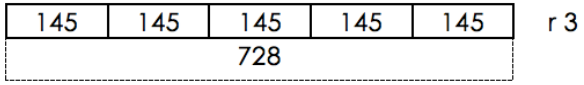
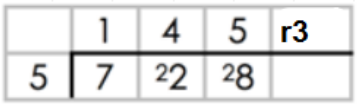
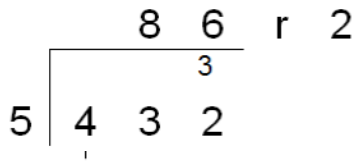
Children to be able to make sense of the place value counters and write calculations to show the process

.
42 ÷ 3
42 = 30 + 12
30 ÷ 3 = 10
12 ÷ 3 = 4
10 + 4 = 14

42 ÷ 3 = 14

$$3\overline{)696}\quad 232$$

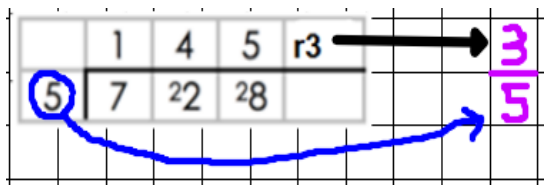
Solving missing number problems: part unknown		 	36 ÷ _____ = 4 We know the whole is 36 We can either say: 'we know there are 4 in each group/part, so how many groups/parts' Or 'we know there are 4 groups/parts in total, so how many in each group/part'?
Year 4 Division			
Short division HTO ÷ O (No regrouping and no remainder) HTO ÷ O (No regrouping and a remainder) HTO ÷ O	Short division using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens.	Draw the counters and cross out any that are regrouped. 	Introduce **short division method.** Begin with divisions that divide equally with no remainder $5\overline{)615}^{\,1\,1}$ = 123 Move onto divisions with a remainder.

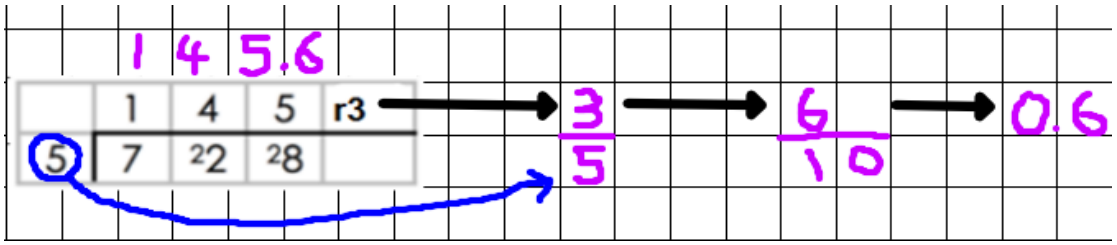
(Regrouping of hundreds into tens) HTO ÷ O (Regrouping of tens into ones) HTO ÷ O (Regrouping of hundreds into tens and tens into ones and a remainder Where there are zeros in the quotient (e.g. 816 ÷ 4 = 204)	4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Bar modelling can support learners when solving problems with division alongside the formal written methods.  728 ÷ 5 = 145 r 3 	 

Year 5 Division

Short division

Revision of **short division method:**





Finally move into decimal places to divide the total accurately.

Remainder expressed as a fraction

Remainder expressed as a simplified fraction

Remainder expressed as a decimal

```
        1 4 . 6
      ---------
      16  21
3 5 | 5 1 1 . 0
```

Year 6 Division

Long division

(2 digit divisors)

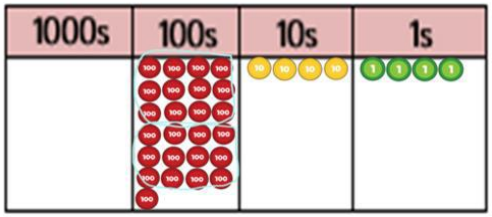
Could introduce long division using place value counters 2544 ÷ 12



We can't group 2 thousands into groups of 12 so will exchange them.

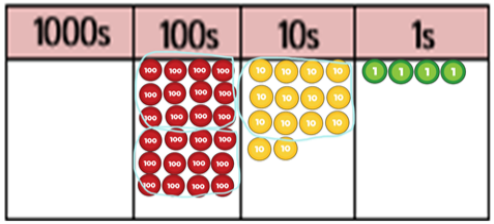
```
      0 2 1 2
    ----------
12 | 2 5 4 4
     2 4
    -------
     1 4
     1 2
    -------
       2 4
       2 4
    -------
         0
```

9382 ÷ 37



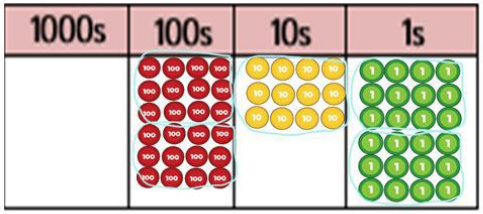
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$



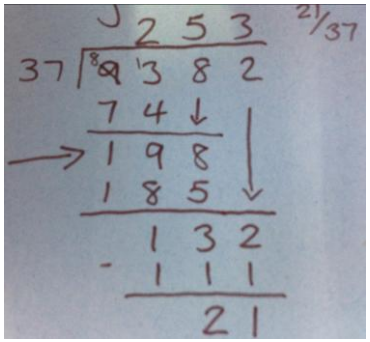
After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$



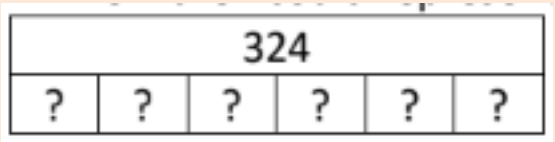
After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$



UKS2 Bar Models

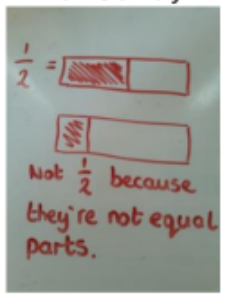
As with multiplication, bar models can be used to help represent and understand the structures of a question but would not be suitable for arithmetic questions such as 324 ÷ 6 if the child is going to 'count up' in 6s as this would be inefficient. Here, we would encourage them to use written methods of division. However, bar models could still be used to show an understanding of worded problems e.g.
There are 324 chairs to put in the hall. The headteacher wants to put them in 6 rows. How many chairs will be in each row?
Children could represent it as:



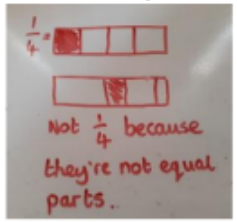
Fractions – representing fractions

Year 1

- *recognise, find and name a half*



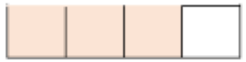
- *recognise, find and name a quarter*



Emphasis is placed on understanding fractions are parts of a whole.

Year 2

- *recognise, find, name and write fractions* $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ *including unit and non-unit.*



$\frac{3}{4}$ The whole has been split/divided into 4 parts and we are looking at 3 parts OR there are 3 shaded.

Year 3

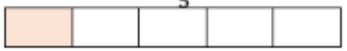
- *count up and down in tenths*
- *recognise that tenths arise from dividing an object into 10 equal parts*

$\frac{3}{10}$



- *recognise and use fractions as numbers: unit and non-unit fractions with small denominators*

Unit fraction: $\frac{1}{5}$



Non unit fraction: $\frac{3}{5}$



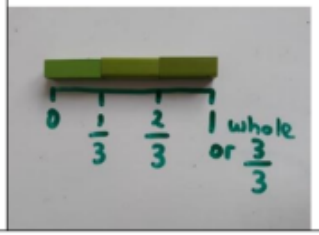
- *making the whole*



1 whole is the same as $\frac{\square}{\square}$

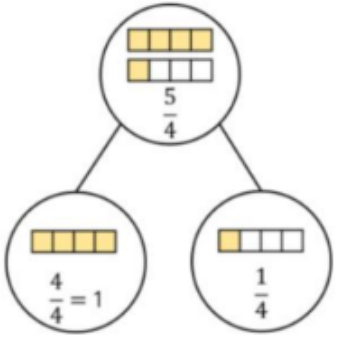
[White Rose Y3 planning document]

- *fractions on a numberline*



Year 4

- *Fractions greater than 1 whole*



There are ___ quarters altogether.

___ quarters = ___ whole and ___ quarter.

[White Rose Y4 planning document.]

Year 5 | Year 6

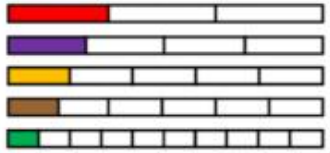
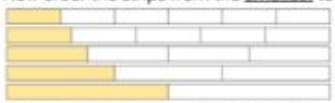
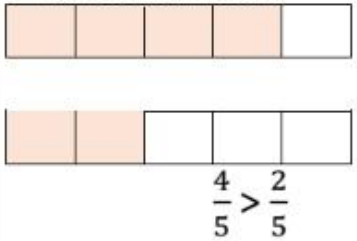
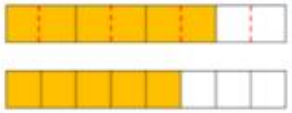
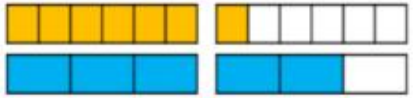
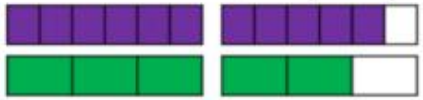
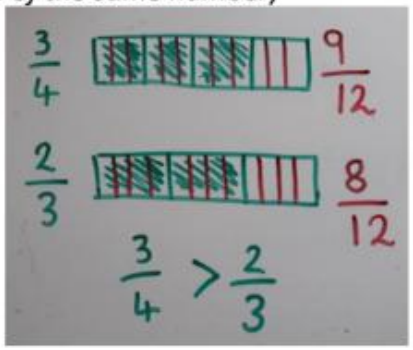
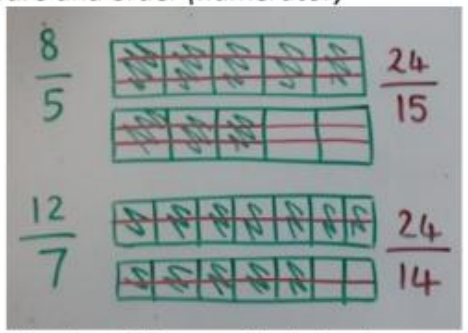
- *convert from improper fractions to mixed numbers*



[White Rose planning document]

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Fractions – comparing fractions

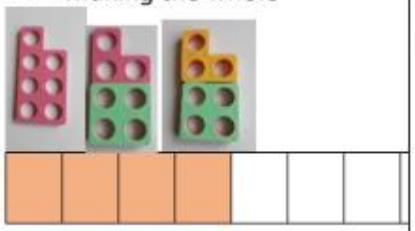
Year 2	Year 3	Year 4	Year 5	Year 6
Teacher might discuss that halves are bigger than thirds and quarters by showing bar models. Though this should be done with caution so that children do not think ½ is always bigger; it is dependent on the size of the whole.	Use fraction walls where possible and Cuisenaire rods to support understanding. • *compare and order unit fractions (same numerator)* Use >, < or = to compare the fractions.  $\frac{1}{10}$ ◯ $\frac{1}{4}$ $\frac{1}{3}$ ◯ $\frac{1}{6}$ $\frac{1}{5}$ ◯ $\frac{1}{4}$ Now order the strips from the smallest to the largest fraction.  When the numerators are the same, the _____ the denominator, the _____ the fraction. [Y3 White Rose document] • *compare and order fractions with the same denominators*  $\frac{4}{5} > \frac{2}{5}$		• *compare and order fractions less than 1* Use bar models to compare $\frac{5}{8}$ and $\frac{3}{4}$  [Y5 White Rose document] • *compare and order fractions greater than 1* Use bar models to compare $\frac{7}{6}$ and $\frac{5}{3}$  Use a bar model to compare $1\frac{2}{3}$ and $1\frac{5}{6}$  [Y5 White Rose document]	• *compare and order (denominators are not multiples of the same number)*  • *compare and order (numerator)*  These methods will be used to introduce and embed the structure of comparing fractions with different denominators and then children will move to the abstract form of finding common numerators and common denominators by using multiplication.

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Fractions – adding fractions

Year 3

- *making the whole*



$\frac{4}{7}$ and $\frac{3}{7}$ make the whole $\frac{7}{7}$

- *adding fractions*





We can use this model to calculate $\frac{3}{8}+\frac{1}{8}=\frac{4}{8}$

[Y3 White Rose document]

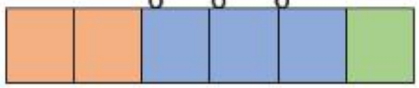
Year 4

- *add two or more fractions*
-

$$\frac{2}{8}+\frac{3}{8}+\frac{1}{8}$$

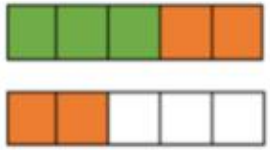


$$\frac{2}{6}+\frac{3}{6}+\frac{1}{6}$$



- *adding fractions and recording the answer using an improper fraction when the answer is greater than 1 whole*

$$\frac{3}{5}+\frac{4}{5}=\frac{7}{5}$$



[Y4 White Rose document]

Year 5



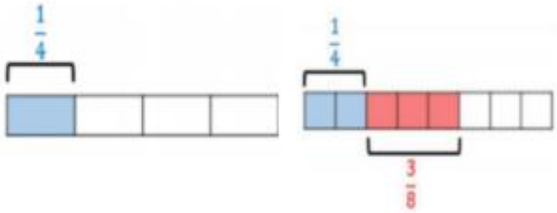
$\frac{3}{5}+\frac{4}{5}=\frac{7}{5}=1\frac{2}{5}$

- *add fractions within one*

$$\frac{1}{2}+\frac{1}{8}=\frac{4}{8}+\frac{1}{8}=\frac{5}{8}$$

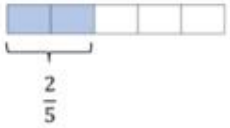


$$\frac{1}{4}+\frac{3}{8}=\frac{2}{8}+\frac{3}{8}=\frac{5}{8}$$



- *add 3 or more fractions*

$$\frac{2}{5}+\frac{1}{10}+\frac{3}{20}$$

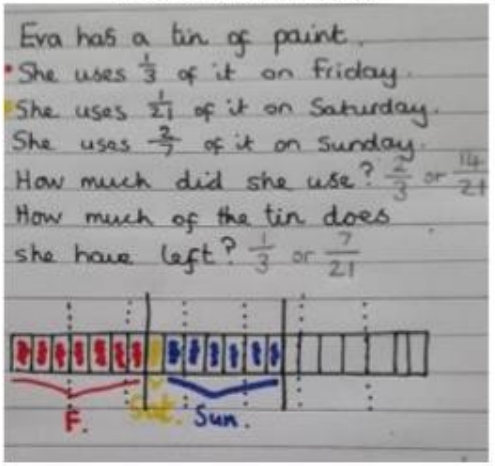


Year 6

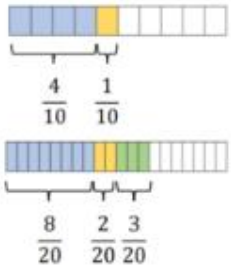
Building on learning from Year 5, children learn to add and subtract fractions within 1 where the children need to find the lowest common multiple in order to find a common denominator (this could be practiced through bar model work as seen in Year 5).

Use the bar model to represent increasingly complex problems where common denominators need to be found.

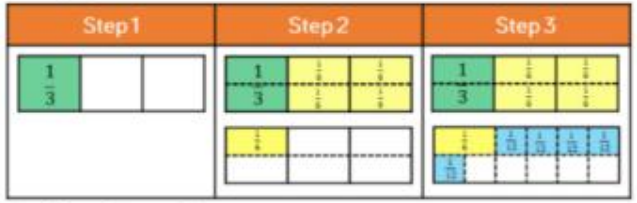
Answers within one:



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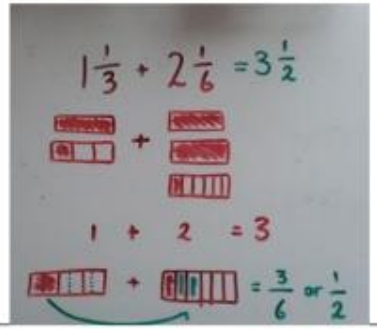
- *add 3 fractions where the answer is greater than 1*



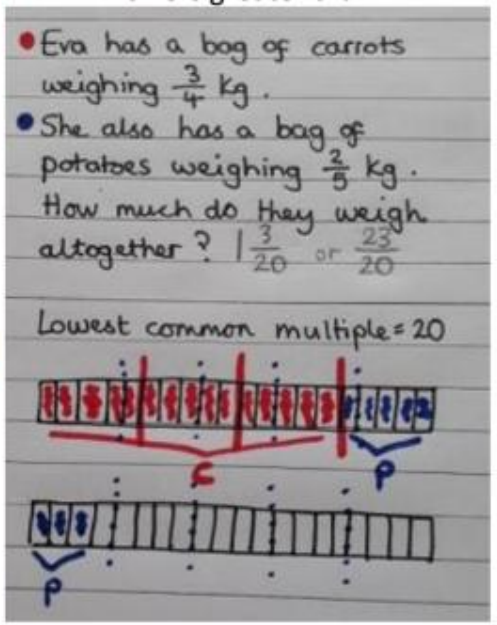
$\frac{1}{3} + \frac{5}{6} + \frac{5}{12} = 1\frac{7}{12}$

[all above images from Y5 White Rose document]

- *add mixed numbers*



Answers greater than 1:

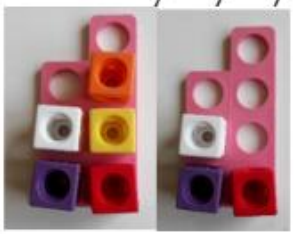


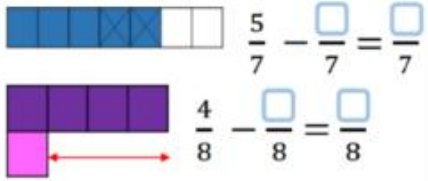
Fractions – subtracting fractions

Year 3

- *subtract fractions with the same denominator within 1 whole*

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$





$$\frac{5}{7} - \frac{\square}{7} = \frac{\square}{7}$$

$$\frac{4}{8} - \frac{\square}{8} = \frac{\square}{8}$$

[Y3 White Rose document]

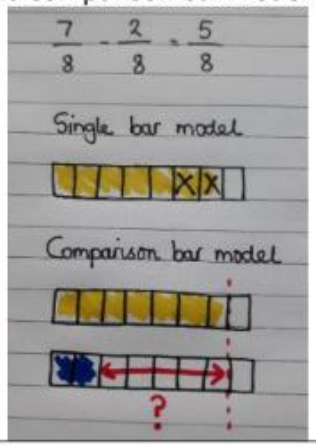
Year 4

- *subtract fractions with the same denominator*

$$\frac{7}{10} - \frac{5}{10} = \frac{2}{10}$$



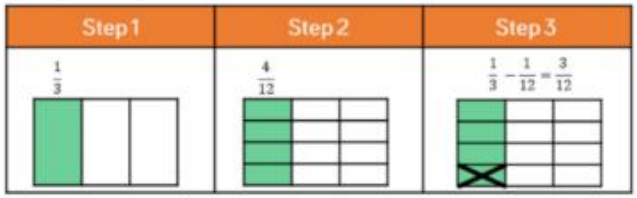
Children should be confident representing the subtraction as both a single bar model and a comparison bar model.



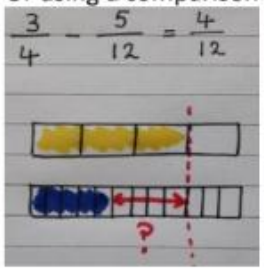
Year 5

- *Subtract fractions with different denominators*

Using a single bar model:

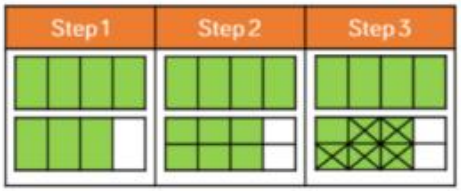


Or using a comparison model:



- *Subtract mixed numbers*

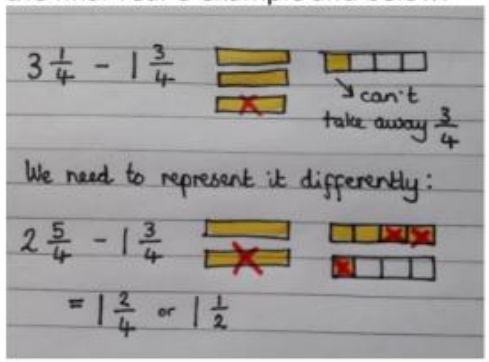
$$1\frac{3}{4} - \frac{5}{8} = 1\frac{1}{8}$$



$$2\frac{3}{4} - \frac{7}{8}$$

Year 6

Continue to embed exchange a whole bar for a bar of fractions as shown in the final Year 5 example and below:

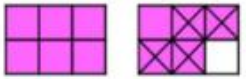


Apply bar modelling representations to help tackle scenario problems.

On Monday she eats $\frac{2}{3}$ of a bag and gives $\frac{4}{5}$ of a bag to her friend.
On Tuesday she eats $1\frac{1}{3}$ bags and gives $\frac{2}{5}$ of a bag to her friend.
What fraction of her sweets does Alex have left?

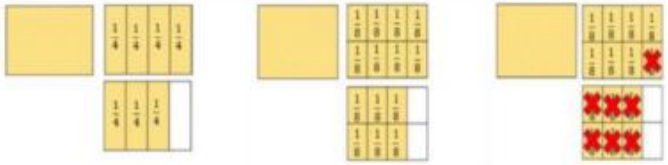
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- *(including subtracting from fractions greater than 1 whole)*



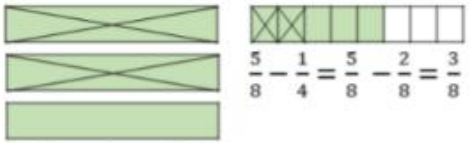
$\frac{11}{6} - \frac{\square}{6}$

[Y4 White Rose document]

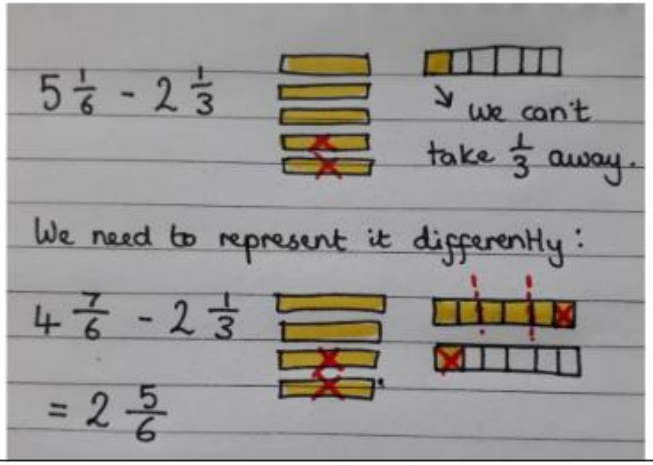


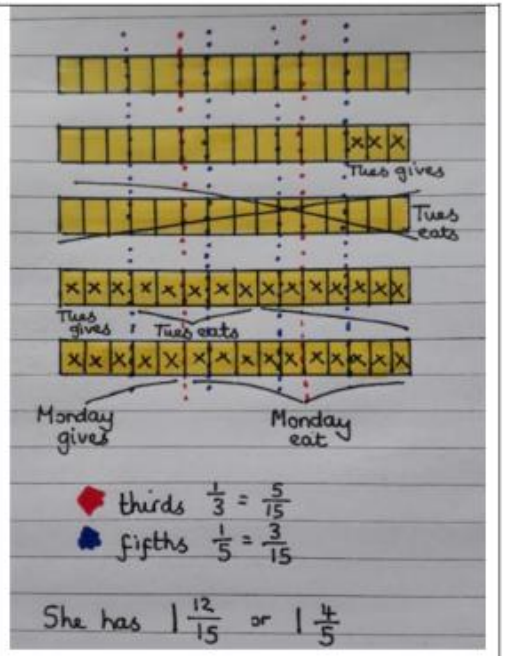
- *Subtract 2 mixed numbers*

$3\frac{5}{8} - 2\frac{1}{4}$



[all above images from Y5 White Rose document]





Fractions – equivalent fractions

Year 2

- Recognise the equivalence of $\frac{1}{2}$ and $\frac{2}{4}$



Additional manipulatives could be used, such as fraction circles.

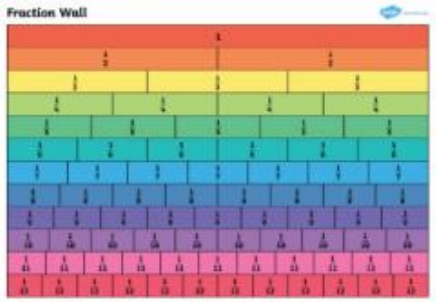
Year 3 | Year 4

- Y3 - recognise and show, using diagrams, equivalent fractions with small denominators
- Y4 - recognise and show, using diagrams, families of common equivalent fractions

Use Cuisenaire rods to investigation equivalent fractions.



Progress to being able to identify equivalent fractions from a pictorial fraction wall where the rows can be treated as 'bars'.



[Y3 White Rose document]

Use the bar model to apply understanding of equivalent fractions through the use of diagrams:



Teddy makes this fraction:





Mo says he can make an equivalent fraction with a denominator of 9



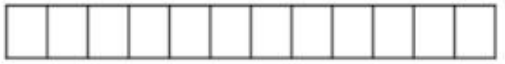
Explain how the diagram shows both $\frac{2}{3}$ and $\frac{4}{6}$



Year 4 > use the bar model as a precursor for recognising the multiplicative relationship between equivalent fractions.

[Y4 White Rose document]

Using the diagram, complete the equivalent fractions.

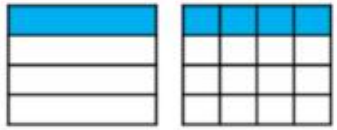


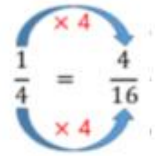
$\frac{1}{4} = \frac{\square}{12}$ $\frac{1}{\square} = \frac{6}{12}$ $\frac{2}{3} = \frac{\square}{12}$ $\frac{5}{12} = \frac{\square}{24}$

Year 5 | Year 6

Revisit concrete and pictorial exploration.

Use models to represent equivalent fractions and illustrate their multiplicative relationship.

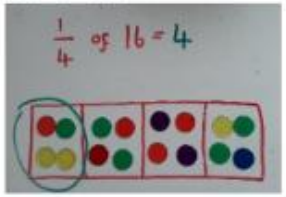
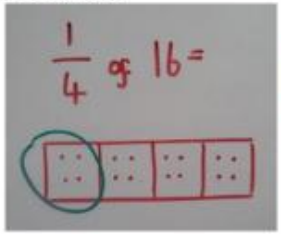
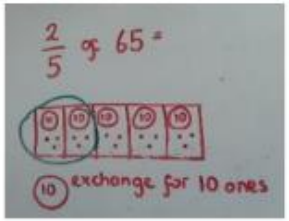
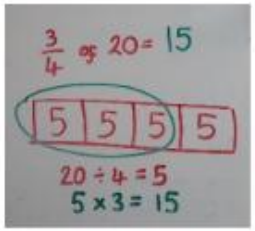
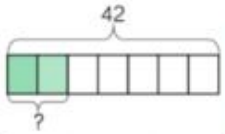
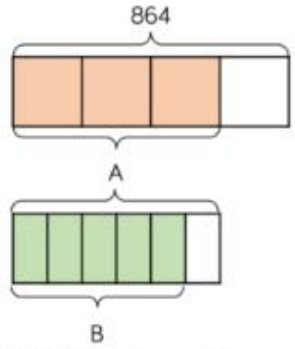
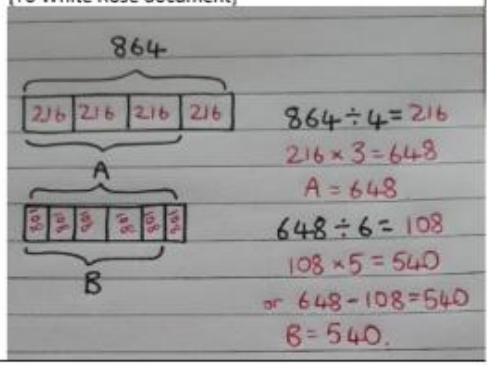


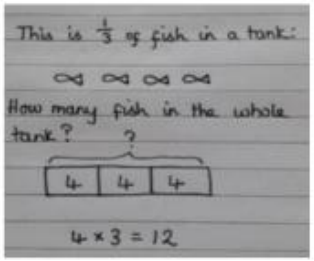
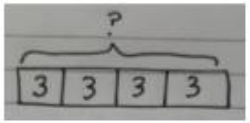
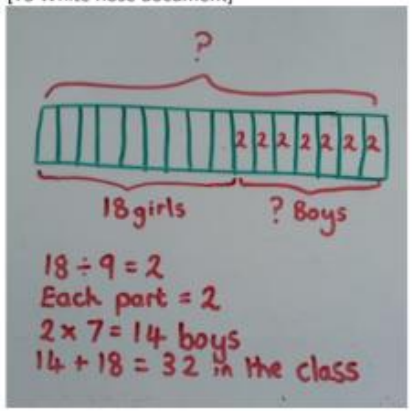
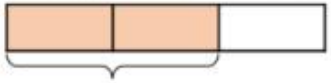
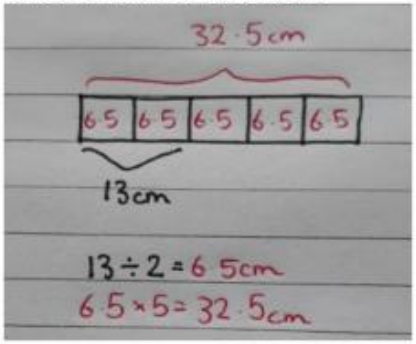


[Y5 white Rose document]

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Fractions – fractions of amounts

Year 2	Year 3	Year 4	Year 5	Year 6
Concrete:  Pictorial:  	Concrete Use place value counters instead of counting in ones when the 'whole' is large as it would be inefficient to use blank counters as 1s. PLACE VALUE COUNTERS Pictorial: Draw out the place value counters.  	Use the same concrete and pictorial methods as Y2 and Y3, depending on the numbers. Progress to using knowledge of times tables to be able to use multiples as the parts.  	Become secure using the abstract method whilst representing this accurately as a bar model e.g. Find $\frac{2}{7}$ of 42.  42 ÷ 7 = 6 6 x 2 = 12 $\frac{2}{7}$ of 42 is 12 [Y5 White Rose document]	Confidently represent problems using bar models to show known and unknown information. Then use the abstract method to calculate the answer. What is the value of A? What is the value of B?  [Y6 White Rose document] 

	• *Solve problems that include calculating the whole quantity.* This is $\frac{3}{4}$ of a set of beanbags.  How many were in the whole set? [Y3 White Rose document] There should be 4 rows, or parts, in total because the denominator is 4. Here, there are 3 equal rows, so another row of the same amount needs to be drawn. 	• *Solve problems that include calculating the whole quantity.* These three squares are $\frac{1}{4}$ of a whole shape.  [Y4 White Rose document] How many squares are in the whole shape? Children should identify that the whole has been divided into 4 parts and one part has 3 squares in it:  They know that each part is equal, so all the other parts also have 3 in them.  3 x 4 = 12 so there are 12 shapes in the whole. This could be introduced practically using squares of paper or cubes and drawing large bar models	• *Solve problems that include calculating the whole quantity* $\frac{7}{16}$ of a class are boys. There are 18 girls in the class. How many children are in the class? [Y5 White Rose document] 	• *Solve problems that include calculating the whole quantity.* Jack has spent $\frac{2}{3}$ of his money. He spent £60, how much did he have to start with?  Eva lit a candle while she had a bath. After her bath, $\frac{2}{5}$ of the candle was left. It measured 13 cm. Eva says:  Is she correct? Explain your reasoning. [Y6 White Rose document] She is incorrect because: 
To find percentages of amounts, the same bar modelling structures could be used as representations just substituting the percentages with fractions.				

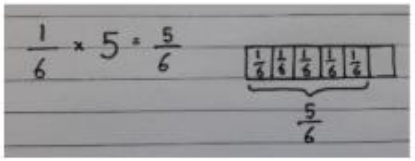
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Fractions – multiplying fractions

Year 5

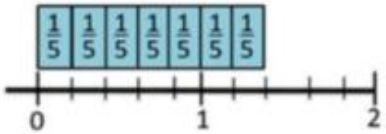
- *multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams*

- *Multiply unit fractions by an integer*



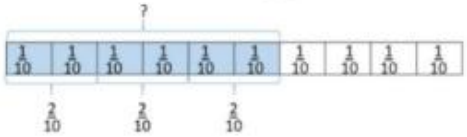
Similar method can be applied on a numberline, particularly if the fraction becomes greater than one.

$\frac{1}{5} \times 7 =$



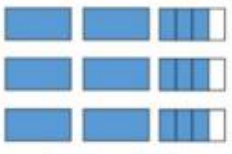
- *Multiply non-unit fractions by an integer*

Use the model to help you solve $3 \times \frac{2}{10}$



- *Multiply mixed numbers by integers*

Partition your fraction to help you solve $2\frac{3}{4} \times 3$



$2 \times 3 = 6$

$\frac{3}{4} \times 3 = \frac{9}{4} = 2\frac{1}{4}$

$6 + 2\frac{1}{4} = 8\frac{1}{4}$

[Y5 White Rose document]

Year 6

- *multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. 1 / 4 × 1 / 2 = 1 / 8)*
- *multiply fractions by integers (build on skills from Year 5)*

Eva partitions $2\frac{3}{5}$ to help her to calculate $2\frac{3}{5} \times 3$



$2 \times 3 = 6$

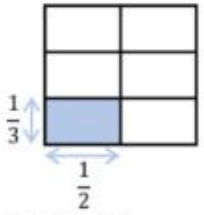
$\frac{3}{5} \times 3 = \frac{9}{5} = 1\frac{4}{5}$

$6 + 1\frac{4}{5} = 7\frac{4}{5}$

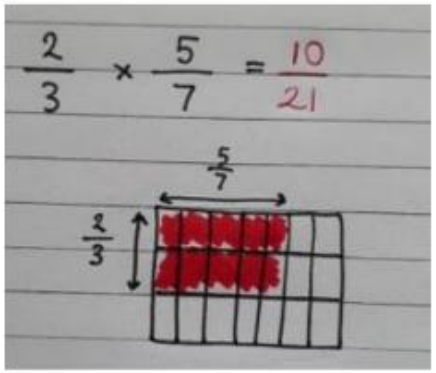


- *multiply fractions by fractions*

$\frac{1}{3} \times \frac{1}{2}$ is the same as $\frac{1}{3}$ of $\frac{1}{2}$



[Y6 White Rose document]



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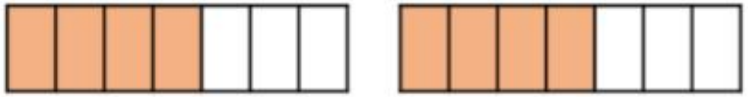
Fractions – dividing fractions

Year 6

- *Divide fractions by integers*

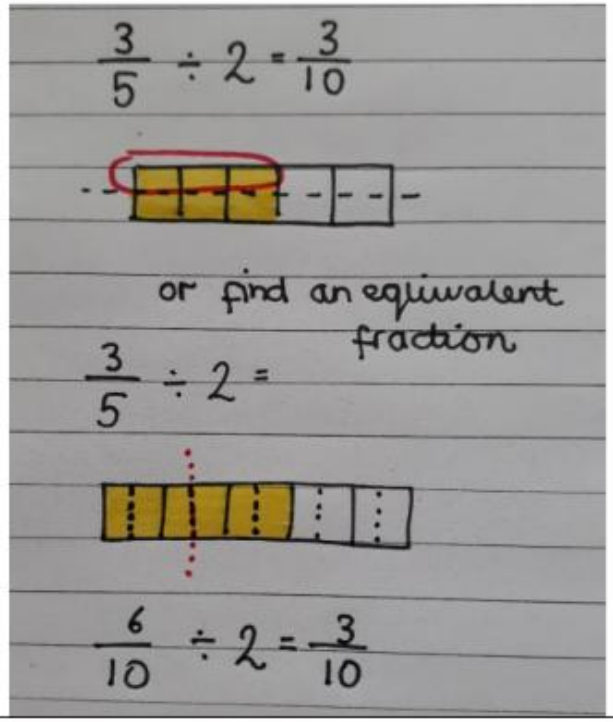
Dividing fractions where the numerator is a multiple of the integer they are dividing by.
Use the sharing method of division.

$\frac{4}{7} \div 4 =$ $\frac{4}{7} \div 2 =$



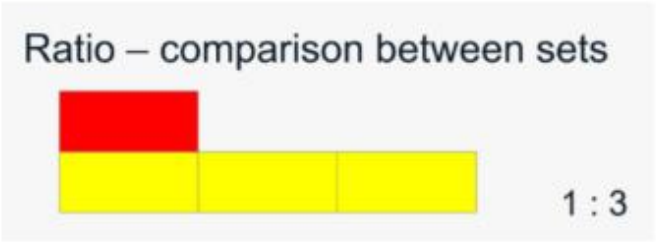
[Y6 White Rose document]

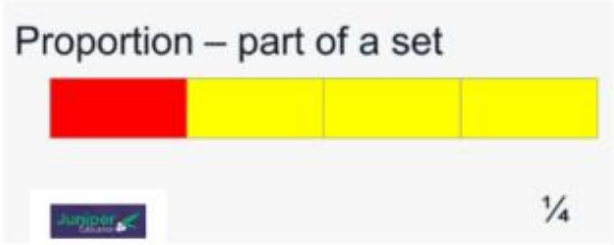
Dividing fractions where the numerator is NOT a multiple of the integer they are dividing by.
Use knowledge of equivalent fractions to create a fraction where the numerator IS a multiple of the integer they are dividing by.



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Ratio and proportion – Year 6

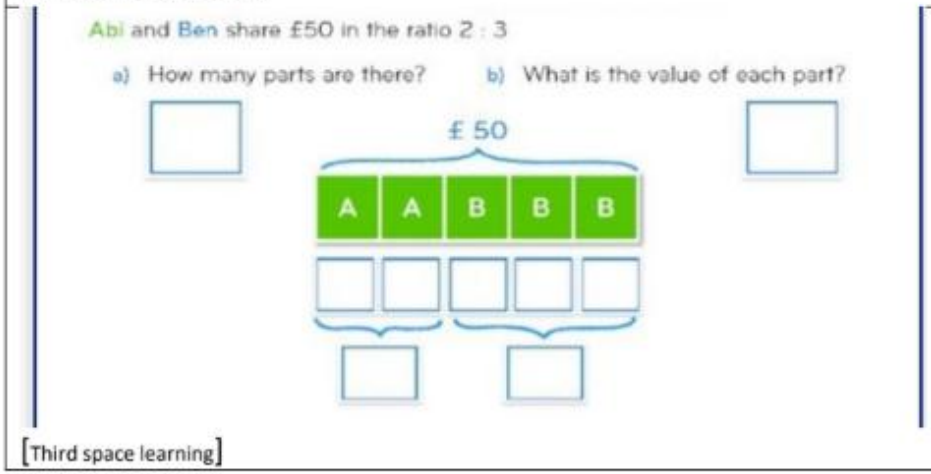




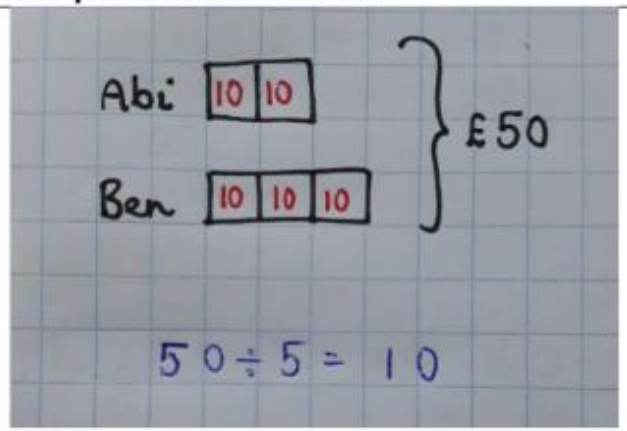
Structures of ratio and proportion are taught before Year 6 (but not explicitly as ratio) through the discussion of equal parts, sharing, and multiplication as repeated addition. The terminology of 'proportion' could be used before Year 6 when talking about fractions of wholes.

Division using ratio could be done using a 'one bar' method or 'comparative bars' (see below). We would encourage children to always use separative bars (the comparative method) because the different parts are easier to see and compare this way. Children who have particularly deep and secure understanding of ratio might be able to work flexibly and effectively using both.

One bar method:



Comparative bars method:

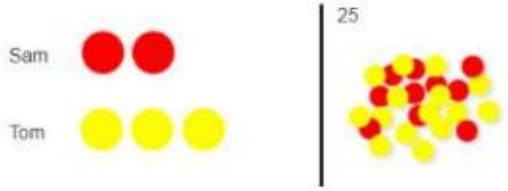


The following examples focus on using bar models to problem solve with ratio problems in Year 6. All of the following examples use ratios comparing two amounts but could easily be adapted for triple ratios e.g. 1:3:4. For examples of how to use bar models for proportion, visit 'fractions of amounts' as proportion means 'part of a whole'.

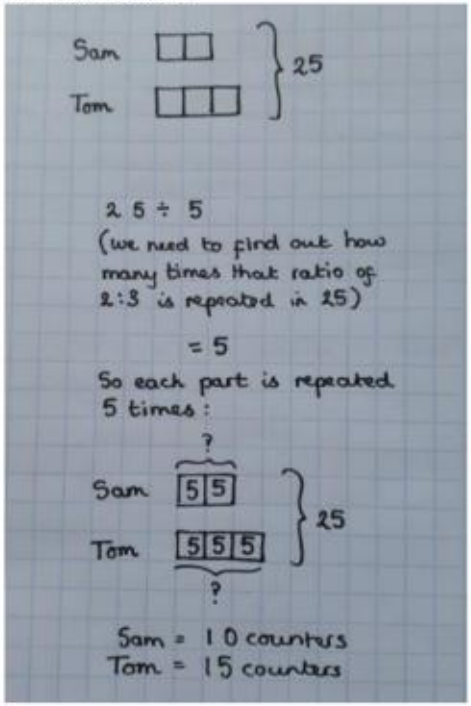
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Finding the value of each part when the whole is known	Finding 'how much more' using ratio.	Finding a whole when a part is given:

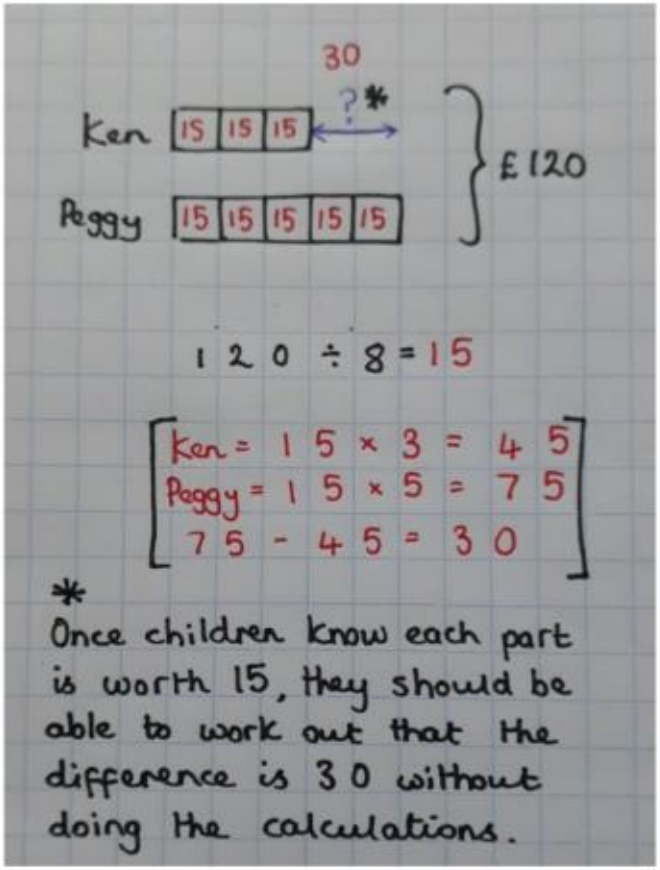
Sam and Tom have football stickers in the ratio of 2 to 3. Altogether they have 25 stickers. How many does Sam have? How many does Tom have?



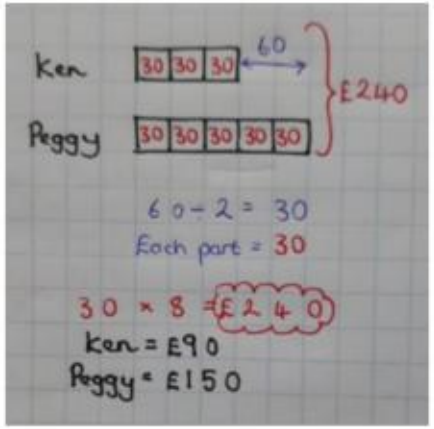
As a bar model:



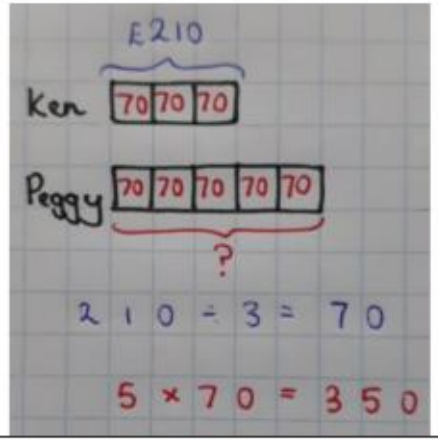
Ken and Peggy share £120 in the ratio 3:5. How much more does Peggy have than Ken?



Ken and Peggy share some money in the ratio 3:5. Peggy gets £60 more than Peggy. How much did they share? How much did they get each?



Ken and Peggy share some money in the ratio 3:5. Ken has £210 pounds. How much does Peggy have?



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Algebra

Defined as: knowing and applying the rules of calculation to find unknown variables and patterns.

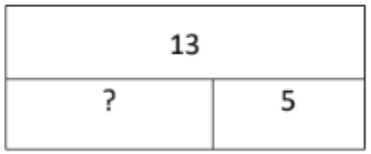
Years 1 & 2

Use bar models to solve missing number questions

e.g.

___ + 5 = 13.

Discuss the known and unknown



This will help children develop algebraic thinking regarding 'the unknown value' and also build their understanding of using inverse relationships to support their algebraic problem solving.

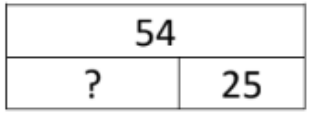
Years 2 & 3

Use bar models to explore the equals sign as a balance point rather than 'on the right'

e.g.

54 = 25 + ___

Discuss the known and unknown parts.

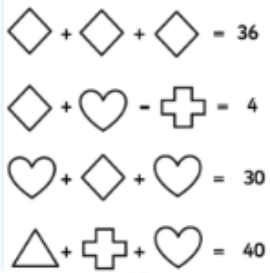


Year 4 & 5

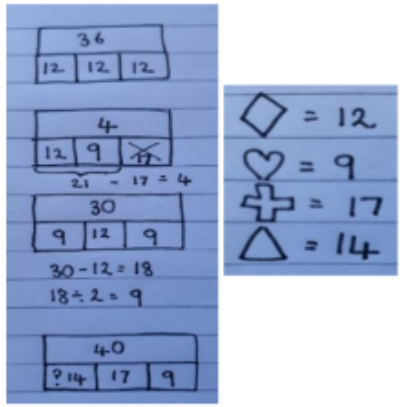
Use bar models to help solve picture problems using the four operations.

e.g.

Work out the value of each shape



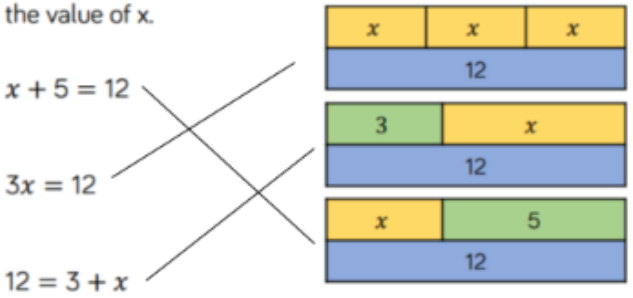
[Classroom secrets example]



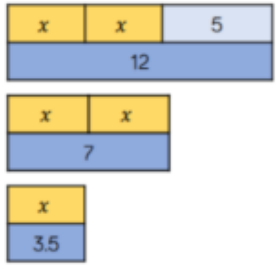
Year 6

Represent algebraic expressions using bar models and use the structure to help work out the answers.

Match each equation to the correct bar model and then solve to find the value of x.



[Y6 White Rose document]



$2x + 5 = 12$

Remove 5 from both sides of the equation (balance the sides).

$2x = 7$

Half x

$x = 3.5$

[Y6 White Rose document]

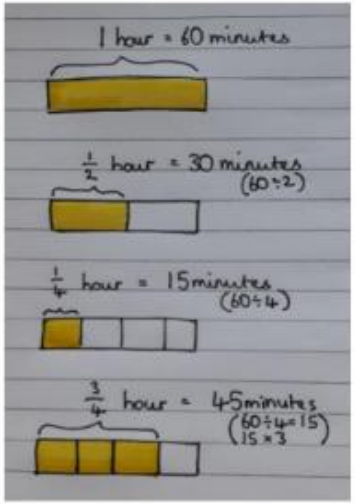
Children should not become reliant on using bar models to solve equations. The most efficient method is to solve them algebraically.

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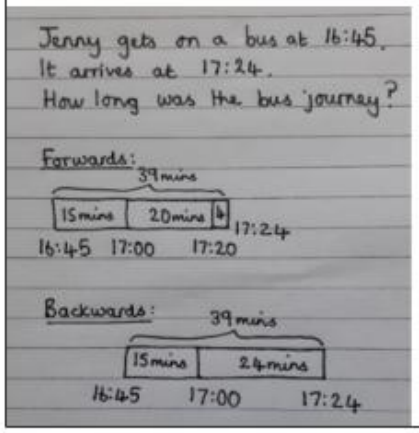
Measurement: Time

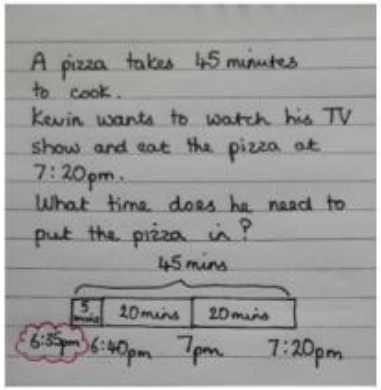
Year 3

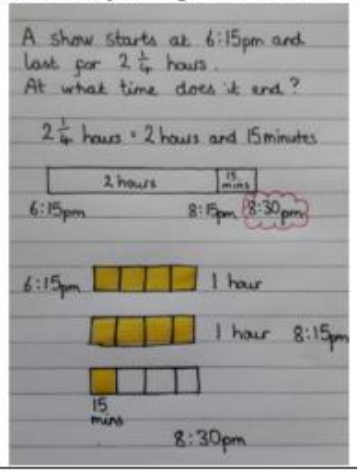
Use bar models to help understand half and quarter hours and their relationship to being a fraction of 60 minutes.



Solve duration problems including calculating the duration or finding start and end times.



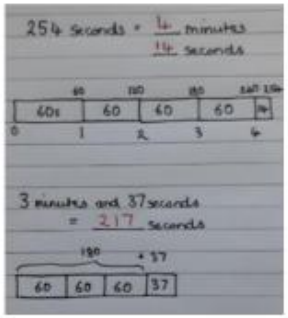


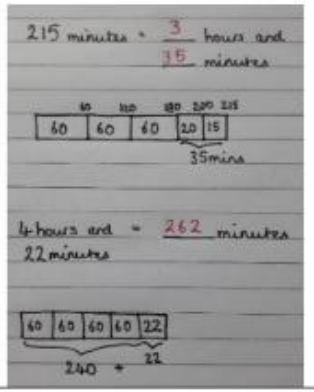


Year 4

Use bar models to help represent conversion between:

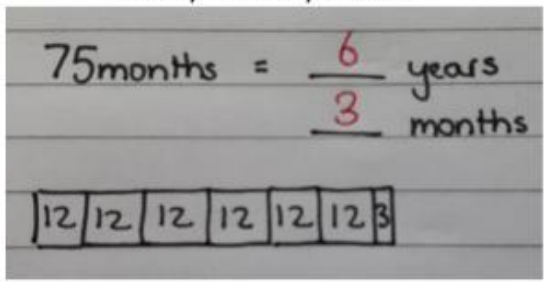
- Hours and minutes, based on the understand that 1 hour is 60 minutes
- minutes and seconds, based on the understanding 1 minute being 60 seconds.

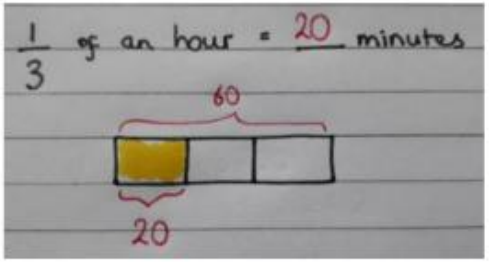


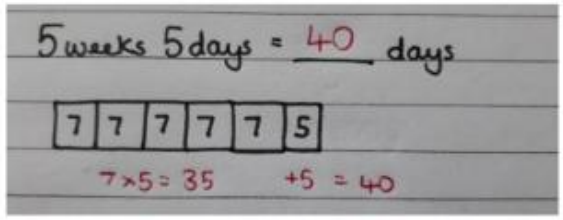


Year 5 | Year 6

Use bar modelling skills learned in Years 3 and 4 and apply these to reading time tables where durations need to be found or measures need to be converted between hours/minutes/seconds.





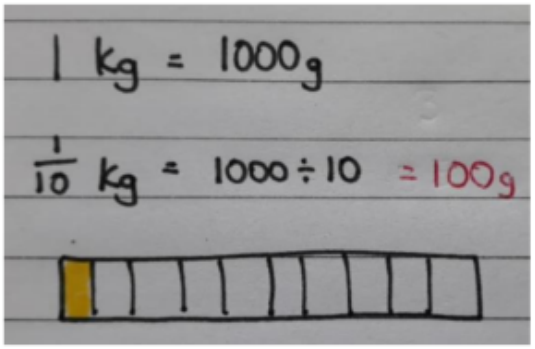


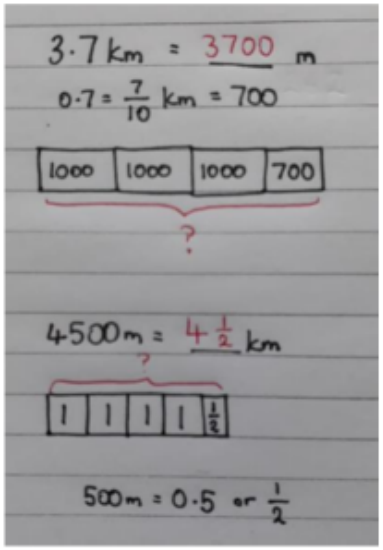
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Measurement: converting units

Year 5

Children will need to be secure in their conversion between fractions and decimals to use these representations accurately.





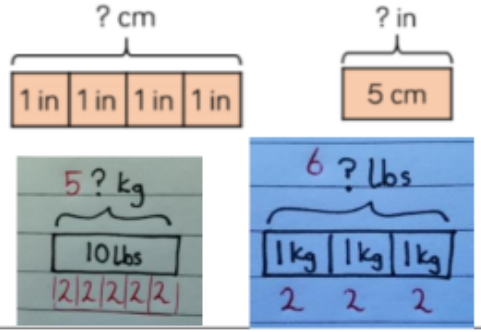
These representations can be applied to mass, length and capacity (including millimetres/milligrams) as a precursor for the abstract method of calculating the conversions.

- *Converting imperial units*

One inch is approximately 2.5 centimetres
1 inch ≈ 2.5 cm

1 kilogram is approximately 2 pounds
1 kg ≈ 2 lbs

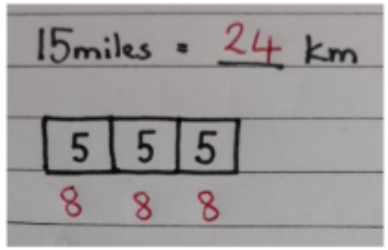
[Y5 White Rose document]



Year 6

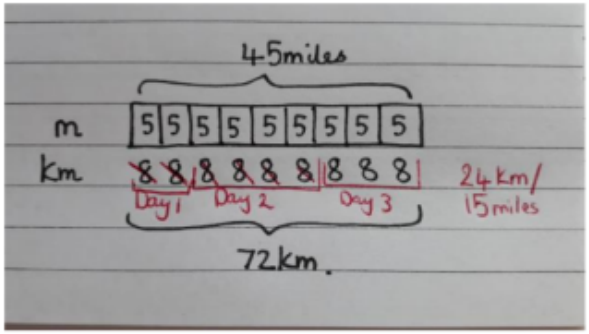
5 miles ≈ 8 kilometres

[Y6 White Rose document]



Applying conversions to problems.

Mo cycles 45 miles over the course of 3 days. On day 1, he cycles 16 km. On day 2, he cycles 10 miles further than he did on day 1 How far does he cycle on day 3? Give your answer in miles and in kilometres.
[Y6 White Rose document]



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Reviewed by Iliana Milios	Autumn Term 2026		
Next Revision (Please highlight as appropriate)	Annual	Bi-annual	Tri- annual
To be reviewed	Autumn Term 2027		